HIGHER SCHOOL CERTIFICATE EXAMINATION 1989
MATHEMATICS - 2/3 UNIT

<u>Direction to Candidates</u>

Time allowed - Three hours (includes reading time)

All questions may be attempted. All questions are of equal value. All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.

Standard integrals are provided; approved calculators may be used.

QUESTION 1

(a) Find correct to 2 decimal places (i) $(2.7)^5$; (ii) $\frac{4.6 - 5.9}{4.6 \times 2.3}$.

(b) Given that $S_n = \frac{n}{2}[2a + (n-1)d]$, find S_n when $n = 103$, $a = 5$ and $d = 1.2$.

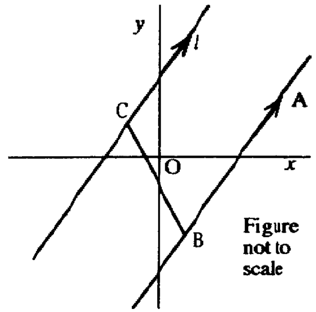
(c) Find the coordinates of the midpoint of the interval AB, where A = (2, 5) and B = (-4, 4).

(d) Solve the equation $4(x - 5) = 3 - 2(x - 1)$.

(e) At a shoe sale, all shoes are to be sold at a discount of 15% off the marked price. What is the cost of a pair of shoes with marked price \$79.95?

QUESTION 2

The line l passes through C(-1, 2) and has equation $y = 2x + 4$.
The point B has coordinates (1, -6) and the line AB is parallel to l.



(i) Copy the diagram and next to the points B and C write their coordinates

(ii) Find the length of the interval BC.

(iii) Write down the slope of the line l and use your calculator to find the angle l makes with the x-axis. Give your answer to the nearest degree.

(iv) Show that AB has equation $y = 2x - 8$.

(v) If P is a point which lies on AB and on the line $y = 2$, find the coordinates of P.

(vi) Find the length of PC.

(vii) On your diagram, draw the line PC and the perpendicular from B to PC.

(viii) Find the area of the triangle PBC.

QUESTION 3

(a) Differentiate (i) $4x^3 - 7 + \frac{5}{x}$; (ii) $x \cos x$.

(b) Find $\int (x^2 + \frac{2}{x})\, dx$.

(c) Evaluate $\int_0^1 (e^{3x} + 1)\, dx$.

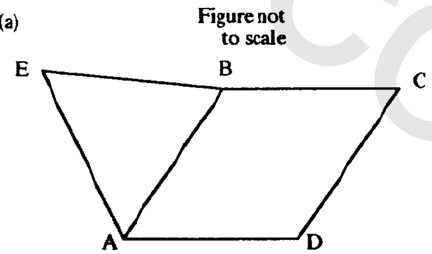
(d) A sales team sells 1200 calculators in its first month of operation. They plan to increase their sales by 150 calculators each month. How many calculators do they plan to sell:

(i) in the last month of the second year of operation;

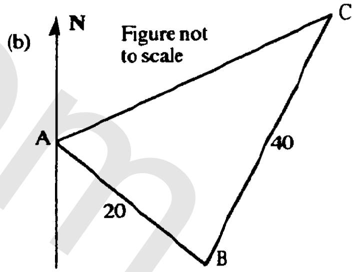
(ii) over the entire two year period?

QUESTION 4

(a)



(b)



(a) ABCD is a rhombus with $\angle BCD = 48°$. ABE is an equilateral triangle.

(i) Draw a neat sketch showing this information.

(ii) Find the size of $\angle EAD$, giving reasons for your answer.

(iii) Find the size of $\angle EDA$, giving reasons for your answer.

(b) Two geologists on a large flat mining claim drive 20 km from a point A on a bearing of 150°T to point B. They then drive 40 km on a bearing of 020°T to point C.

(i) Copy the given diagram and show that $\angle ABC = 50°$.

(ii) Use the cosine rule to find the distance of point C from point A to the nearest kilometre.

(c) In a large school, the student population is 42% male and 58% female. Two students are selected at random to take part in a survey. Find correct to two decimal places, the probability that:

(i) both are females;

(ii) both are of the same sex;

(iii) they are of different sexes.

QUESTION 5

(a) A geometric series has second term 6 and the ratio of the seventh term to the sixth term is 3.

(i) Find the common ratio r.

(ii) What is the first term a ?

(ii) Calculate the sum of the first 12 terms.

(b) What is the volume of the solid of revolution formed by rotating the curve $y = \sec x$ about the x-axis, for $0 \le x \le \frac{\pi}{3}$?

(c) A particle moves in a straight line so that its velocity v in metres per second at time t is given by $v = 4 - 2t$. At time $t = 0$, the particle is at $x = 1$.

(i) Find the displacement x of the particle as a function of t.

(ii) When is the particle at rest and what is its acceleration at that time?

(iii) Find the distance the particle travels in the first 4 seconds.

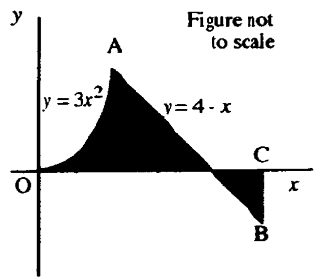
QUESTION 6

(a) Solve the simultaneous equations $4x - y = 3$, $10x + 3y = 2$.

(b) (i) Sketch the parabola P, whose focus is the point (2, 5) and whose directrix is the line $y = -3$. Indicate on your diagram the vertex and its coordinates.

(ii) Find the equation of P.

(c) The shaded region OABC is bounded by the lines $x = 0$, $x = 5$, the curve $y = 3x^2$, the line $y = 4 - x$ and the x-axis, as in the diagram.



(i) Show that A has coordinates (1,3).

(ii) What is the area of the shaded region OABC?

QUESTION 7

(a) (i) Find the equation of the tangent to the curve $y = 3\ln x + 2$ at the point where $x = 1$.

(ii) Find the slope of the normal to the curve at the point where $x = 1$.

(b) The rate of emission E, in tonnes per year of chloro-fluorocarbons (CFCs) in Australia from 31 October 1989 is given by $E = 100 + \left(\frac{40}{1+t}\right)^2$, where t is the time in years.

(i) What is the rate of emission E on 31 October 1989?

(ii) What is the rate of emission E on 31 October 1998?

(iii) What value does E approach as the years pass by?

(iv) Draw a sketch of E as a function of t.

(v) Calculate the total amount of CFCs emitted in Australia during the years 1989 to 1998.

QUESTION 8

(a) Solve the equation $2\cos x = \sqrt{3}$, where $0 \le x \le 2\pi$.

(b) Consider the function defined by $f(x) = e^{2x}(1 - x)$, where $-3 \le x \le 1$

(i) Copy and complete the table of values.

Give values correct to two decimal places.

x	-3	-2	-1	0	1
$f(x)$	0.01	0.05			

(ii) Differentiate $f(x)$ and hence show that the function has only one stationary point.

(iii) Sketch the curve $y = f(x)$ for $-3 \le x \le 1$.

(iv) Without further use of calculus, indicate on your diagram where the curve is concave down.

(v) Using the trapezoidal rule with five function values, approximate the area under the curve $y = f(x)$ for $-3 \le x \le 1$.

(vi) From your diagram, decide whether this approximation is an over-estimate or an under-estimate of the true value of the area under the curve. Give a brief reason.

QUESTION 9

(a) (i) Sketch the function $y = x^2$, $0 \le x \le 2$, and write down its domain and range.

(ii) If y in (i) is part of an odd function $f(x)$ defined for $-2 \le x \le 2$, sketch on a second diagram the function $f(x)$.

(iii) Write down the value of $\int_{-2}^{2} f(x)\,dx$, giving a brief reason for your answer.

(b) Consider the function whose derivative is given by $\frac{dy}{dx} = x^2(2x-1)(x-1)$. Determine the nature of the stationary point at $x = 0$.

(c) ABCD and DEBF are two congruent rectangles with sides 3 and 7 units as in the diagram.
(AB = DF = 7, AD = DE = 3)

(i) Show that $AT = \frac{20}{7}$

(ii) Find the area of the figure DWBT.

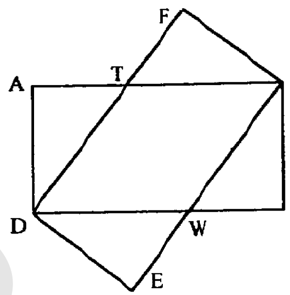


Figure not to scale

QUESTION 10

(a) In the Jackpot Lottery, the probability of the Jackpot prize being won in any draw is approximately 1 in 50.

(i) What is the probability that the Jackpot prize will be won in each of three consecutive draws?

(ii) How many consecutive draws must be made for it to be 99% certain that a Jackpot prize will have been won?

(b) A cam is formed with cross-section as shown in figure. The cross section consists of a semi-circle AXC centre O and radius $\frac{r}{2}$ and a sector ABC of radius r, centre A and angle θ.

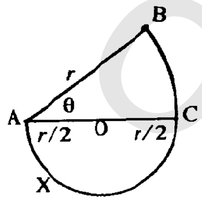


Figure not to scale

(i) What is the perimeter ABCX of the cam in terms of r and θ?

(ii) If the area of the cross section of the cam is 1 square unit, show that the perimeter P is given by $P = \frac{2}{r} + r(1 + \frac{\pi}{4})$.

(iii) Show that the least perimeter occurs when $r^2 = \frac{8}{\pi + 4}$ and calculate the value of θ to the nearest degree.