HIGHER SCHOOL CERTIFICATE EXAMINATION 1988
MATHEMATICS - 2/3 UNIT

Direction to Candidates

Time allowed - Three hours (includes reading time)

All questions may be attempted. All questions are of equal value. All necessary working should be shown in every question. Marks may not be awarded for careless or badly arranged work.

Standard integrals are provided; approved calculators may be used.

QUESTION 1

(a) Find, correct to two decimal places, the value of $\dfrac{(3.24)^2}{5.73 - 2.84}$.

(b) Simplify the expression $4x - 3(x + 5)$.

(c) The base length x, of a square pyramid of volume V and perpendicular height h, is given by $x = \sqrt{\dfrac{3V}{h}}$.

Find x, correct to two decimal places, if $V = 750$ and $h = 8.45$.

(d) Solve the equation $3x^2 - 6x + 1 = 0$ giving each solution correct to two decimal places.

(e) Solve the equation $\dfrac{2x}{x-5} = \dfrac{3}{5}$.

QUESTION 2

(a) Differentiate (i) $3x^4 - 2x + \dfrac{1}{x^2} + 1$; (ii) xe^{2x}; (iii) $\dfrac{x}{1+x}$

(b) Find, correct to one decimal place, the value of: (i) $\int_0^1 (3x^6 + 1)\,dx$ (ii) $\int_0^{\pi/4} \cos 3x\,dx$

QUESTION 3

Q is the point of intersection of the x-axis and the line l with equation $2x - 3y = 2$.

(a) On a number plane draw the line l, marking on it the point Q.

(b) On your diagram, indicate the point P(4, 2) which lies on l. Draw the line k through P perpendicular to l.

(c) Find the equation of the line k.

(d) Without calculating its coordinates, indicate a point R on k which is one unit from P. Mark the right angle RPQ on your diagram.

(e) Find the distance PQ.

(f) Find the area of the triangle QPR.

(g) On your diagram shade the region given by $x \geq 0$, $2x - 3y \geq 2$.

QUESTION 4

(a) The first three terms of an arithmetic series are 12, 17 and 22.

(i) Find the twenty-fifth term of this series. (ii) Find the sum of the first twenty-five terms.

(b) The displacement x metres from the origin at time t seconds of a particle travelling in a straight line is given by $x = t^3 - 9t$, where $t \geq 0$.

(i) Find the velocity at time t seconds. (ii) Calculate the velocity when $t = 2$.

(ii) Find the time when the particle is stationary.

(c) PQR is an isosceles triangle in which PQ = PR and ∠QPR = 104°. QR is produced to N. QM bisects ∠PQR and RM bisects ∠PRN.

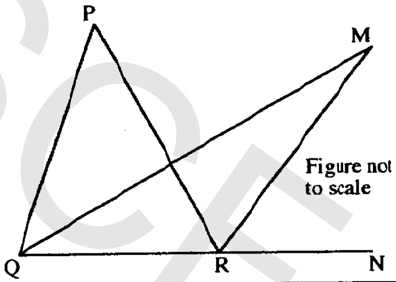


Figure not to scale

(i) In your writing booklet draw a neat sketch and mark on it all the given information.

(ii) Find the size of ∠PQR. Give reasons for your answer.

(iii) Find the size of ∠QMR. Give reasons for your answer

QUESTION 5

(a) ABC is a triangle in which AB = 3 cm, BC = 6 cm and CA = 5 cm, as in the given figure.

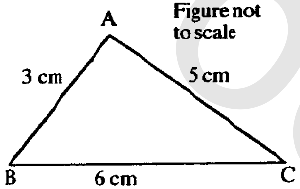


Figure not to scale

(i) Use the cosine rule to calculate ∠BAC to the nearest degree.

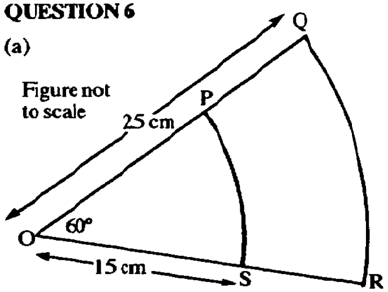
(ii) Calculate the area of the triangle ABC. Give your answer correct to two significant figures.

(b) Find the equation of the tangent to the curve $y = x^2 + \frac{5}{x} - 2$ at the point P(1, 4).

(c) The curve $= f(x)$ has gradient function $\frac{dy}{dx} = 3x^2 - 2x + 1$. The curve passes through the point Q(2, 3). Find its equation.

QUESTION 6

(a)



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(b)

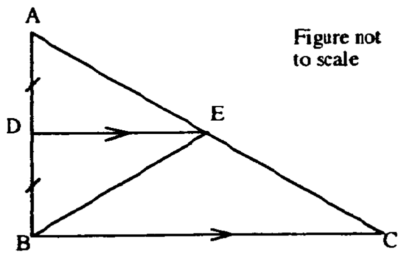

Figure not to scale

(a) PS and QR are arcs of concentric circles with O as centre. Calculate in terms of π:

(i) the area of the shaded region PQRS, (ii) the perimeter of the shaded region PQRS.

(b) The triangle ABC has a right angle at B. D is the midpoint of AB. E lies on AC and DE is parallel to BC.

(i) Copy this diagram into your writing booklet. Prove that ADE is a right angle.

(ii) Prove that triangle AED is congruent to triangle BED.

(iii) Prove that BE = EC.

QUESTION 7

(a) The region bounded by the curve $y = \sqrt{x} - 1$, the lines $x = 2$, $x = 3$ and the x-axis, is rotated about the x-axis. Find the volume of solid of revolution so formed.

(b) Consider the function $f(x) = x - 3\ln x$, where $1 \leq x \leq 7$.

(i) There is one turning point for $f(x)$. Find its coordinates and determine whether it is a maximum or minimum turning point.

(ii) Draw a sketch of the curve $y = f(x)$, where $1 \leq x \leq 7$.

(iii) What is the maximum value of the function $f(x)$, where $1 \leq x \leq 7$?

QUESTION 8

(a) A parabola P has equation $x^2 = 8(4 - y)$.

(i) Draw a neat sketch of P and clearly indicate on it:

(α) the equation of its directrix; (β) the coordinates of its focus;

(γ) the coordinates of all points of intersection of P with the coordinate axes.

(ii) Another parabola Q with equation $x^2 = 8y$ cuts P at A and B. Find the coordinates of A and B.

(iii) Hence calculate the exact value of the area of the region bounded completely by P and Q.

(b) Find all real numbers x which satisfy the equation $x^4 = 4(x^2 + 8)$

QUESTION 9

(a) Farmer Brown wishes to construct three rectangular enclosures, as shown, in which to put pigs and calves. The paddock for the calves is to be six times as long and twice as wide as a pig pen. One pig pen and the calves' paddock have an existing brick wall as a boundary fence as shown. All other fences are to be constructed from 56 metres of wire mesh.

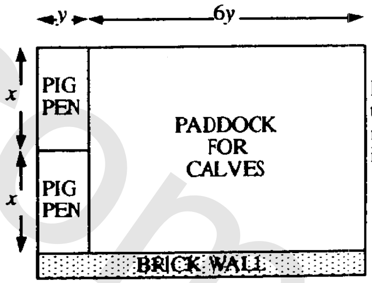

Figure not to scale. All measurements in metres.

(i) Let x metres be the width of a pig pen and y metres be its length.

Show that $y = 7 - \frac{3}{4}x$.

(ii) Hence show that the total area A square metres contained in the three enclosures is given by

$A = 14x(7 - \frac{3}{4}x)$.

(iii) Show that A is a maximum when half the wire fencing has been placed parallel to the brick wall.

(b) (i) A die whose faces are numbered 1, 2, 3, 4, 5, 6 is tossed twice. The sum S of the numbers which appear uppermost on the die is calculated. Find the probability that S is greater than 8.

(ii) It is known that a 4 appears on the die at least once in the two throws. Find the probability that S is greater than 8.

QUESTION 10

(a) The councils in two towns A and B have found that the populations in the towns are given by:

$P_A = 2000e^{-0.02t}$ for town A, $P_B = 1000e^{0.03t}$ for town B, where t is the number of years which have elapsed since January 1st, 1980.

(i) Write down the annual growth rate for B.

(ii) Calculate the instantaneous rate at which the population of B will be increasing at the start of 1995.

(iii) During which year will the population of B become larger than the population of A?

(b) Find, as a relation between k, l and m, the condition for the quadratic equation in x $(k^2 + l^2)x^2 + 2l(k + m)x + (l^2 + m^2) = 0$ to have real roots. Simplify your answer as far as possible.