N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1979
MATHEMATICS - 2 UNIT COURSE

TIME 3 HOURS

Instructions All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1

(i) Differentiate with respect to x: (a) $4x^3 + x\sqrt{x}$ (b) e^{x^2+1} (c) $\frac{2x-1}{3x+2}$

(ii) Find (a) $\int_2^4 (\frac{1}{x} + \frac{1}{x^2})\,dx$ (b) $\int_0^\pi (2\sin x - \sin 2x)\,dx$

(iii) If $\int_0^a (4-2x)\,dx = 4$, find the value of a.

(iv) If $\frac{dy}{dt} = 6t^2 - 2$ and $y = 1$ when $t = 1$, find y when $t = 2$.

QUESTION 2

(i) Write in simplest form the expression $2x - (x-2)$.

(ii) Solve each of the following: (a) $-3 \le 1 - x < 4$ (b) $|x+1| = 4$

(iii) Rationalise the denominator, and express in simple surd form: $\frac{2\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(iv) A function is defined by the rule: $f(x) = \begin{cases} 0 \text{ if } x \le -2 \\ -1 \text{ if } -2 < x < 0 \\ x \text{ if } x \ge 0 \end{cases}$

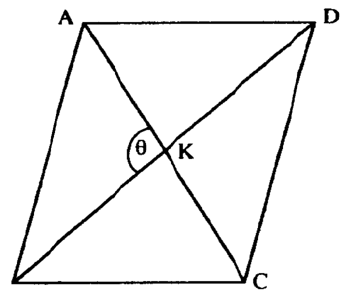
Find (a) $f(-2) + f(-1) + f(0)$ (b) $f(a^2)$

QUESTION 3

(i) Find the exact value of $\frac{2}{15} - \frac{275}{1000}$ as a fraction in its lowest terms.

(ii) Express $\frac{x+1}{x^2-x} - \frac{x-1}{x^2+x}$ as a fraction in its lowest terms.

(iii) ABCD is a parallelogram (as shown in the diagram) in which the lengths of AB and AD are 3 cm and 1 cm respectively and the angle ABC is 60°; also the diagonals cut at K and the angle AKB is denoted θ.



Calculate the exact values (giving answers as rational numbers or surds) of:

(a) the length of the diagonal AC;

(b) the length of the diagonal BD;

(c) cos θ

QUESTION 4

The point Q(-2, 1) lies on the line k whose equation is $9x - 2y + 20 = 0$. The point R(4, -2) lies on the line l whose equation is $3x + y - 10 = 0$.

(i) Show that k and l intersect at a point P on the y-axis.

(ii) Find the equation of the line m which joins Q and R.

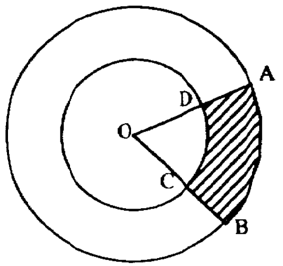
(iii) Show, by shading on a sketch (not on graph paper), the region defined by the three inequalities $9x - 2y + 20 \ge 0$, $3x + y - 10 \le 0$, $x + 2y \ge 0$.

(iv) Find, as a surd, the perpendicular distance from P to m.

(v) Hence, or otherwise, find the exact value of the area of the triangle bounded by the three lines k, l, m.

QUESTION 5

(i) The diagram shows two concentric circles centre O and radii 20 cm and 10 cm respectively. ODA, OCB are straight lines and the angle between OA and OB is 60°. Given $\pi = 3.142$, find, correct to 3 significant figures:



(a) the perimeter of the shaded region ABCD;

(b) the area of the shaded region ABCD.

(ii) (a) Find the focus and directrix of the parabola $x^2 = 4y$

(b) A line is drawn through the focus of the above parabola parallel to the x-axis. The region bounded by this line and the parabola is rotated about the y-axis. Find the volume of the solid of revolution.

(iii) Find the values of m for which the equation $x^2 + (m-2)x + 4 = 0$ has

(a) equal roots (b) no real roots

QUESTION 6

(i) Find the minimum value of $x + \frac{900}{x}$ for $x > 0$, giving reasons for your answer.

(ii) A cargo service operates by running a ship between port A and port B at a constant speed of v kilometres per hour.

For a given v, the cost per hour of running the ship is $9000 + 10v^2$ dollars.

Find the value of v which minimizes the cost of the trip.

QUESTION 7

(i) Two ordinary dice, with numbers 1 to 6 on their faces, are thrown. What is the probability that

(a) they both show 6? (b) they show a 1 and a 6?

(c) at least one shows a 1? (d) they show a total of six?

(ii) On a destroyer there are two lines of defence against aircraft attack. These are a surface-to-air missile and a 15 mm rapid-firing gun. The probability of success in hitting an attacking aircraft with each line of defence is respectively 0.9 and 0.8.

Find the probability of hitting an attacking aircraft before it penetrates both defences.

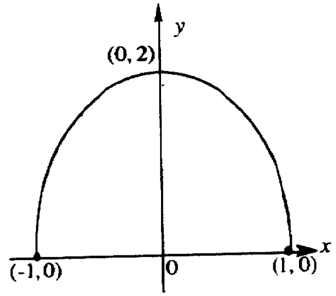
QUESTION 8

(i) Draw sketches (not on graph paper) of

(a) $y = 3\cos\frac{1}{2}x$ from $x = -\pi$ to $x = \pi$

(b) $y = \cos \pi x$ from $x = -1$ to $x = 1$

(ii) An ornamental arch window 2 metres wide and 2 metres high is to be made in the shape of an arc of either a cosine curve or a parabola, as illustrated on the axes.



(a) If the arch is made in the shape of the curve $y = 2\cos\frac{\pi}{2}x$, find the area of the window. (Your answer may be left in terms of π.)

(b) If the arch is made the shape of an arc of a parabola, find the equation of the parabola and the area of the window.

QUESTION 9

(i) Given $\log_2 3 = 1.158\,51$, find correct to two decimal places: (a) $\log_2 9$ (b) $\log_2 12$

(ii) Show that the tangents to the graphs of $y = \log_e x$ and $y = \log_e 2x$ at the points where $x = 1$ are parallel.

(iii) A vertical line $x = a$ is drawn to cross the two graphs $y = \log_e x$ and $y = \log_e 2x$ at points A and B. Find the distance AB and show it is constant (that is, it does not depend on the value of a).

(iv) A horizontal line is drawn to cross the two graphs $y = e^x$ and $y = \frac{1}{2}e^x$ at points C and D. Show that the distance CD is constant (that is, does not depend on the position where the horizontal line is drawn).

QUESTION 10

(i) The first three terms of an arithmetic series are 50, 43, 36.

(a) Write down a formula for the n-th term.

(b) If the last term of the series is -27, how many terms are there in the series?

(c) Find the sum of the series.

(ii) A loan of $1000 is to be repaid by equal annual instalments, repayments commencing at the end of the first year of the loan. Interest, at the rate of 10 per cent, is calculated each year on the balance owing at the beginning of that year, and added to that balance.

If the annual instalment is P dollars, prove that

(a) the amount owing at the beginning of the second year of the loan is (1100 - P) dollars;

(b) the amount owing at the beginning of the third year of the loan is (1210 - 2.1P) dollars;

(c) if the loan (including interest charges) is exactly repaid at the end of n years; then

$$P = \frac{100}{\left(1 - \frac{1}{(1.1)^n}\right)}$$