N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1978
MATHEMATICS - 2 UNIT COURSE

TIME 3 HOURS

Instructions   All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1

(i) Find

(a) tan 220° (correct to 4 decimal places);

(b) the value of θ (to the nearest degree) if $\cos\theta = -0.7$ and $180° \le \theta \le 360°$;

(c) the values of x (to the nearest degree) in the domain $0° \le x \le 360°$ for which $\sin x > 0.4$.

(ii) Show that $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$

(iii) By expressing $\frac{4}{2+\sqrt{5}} - \frac{1}{9-4\sqrt{5}}$ in its simplest form show that it is a rational number.

(iv) (a) Find $\{x : |x-2| \le 1\}$

(b) Evaluate $\frac{|x|}{x}$ where $x \ne 0$

QUESTION 2

(i) Find the exact value (as a rational number in its lowest terms) of:

(a) $\frac{1}{2}+\frac{2}{3}-\frac{4}{7}-\frac{5}{14}$

(b) $\frac{A^4C}{B^4}$ where $A = (\frac{2}{3})^2$, $B = (\frac{4}{3})^4$, $C = (\frac{8}{3})^7$

(ii) If the volume of one litre of a certain liquid decreases to V litres after n days according to the formula $V = (1-r)^n$, find the value of r (to three significant figures) if $V = 0.31$ after five days.

(iii) In the triangle ABC the side BC has length 4 cm and the angles at B, C are 60°, 70° respectively. Determine (to three significant figures):

(a) the length of AB

(b) the length of AP where P is the midpoint of BC

QUESTION 3

(i) The series $\frac{1}{3}-\frac{1}{6}+\frac{1}{12}-\dots$ is geometric. Find the sum of the first ten terms. (Give the answer as a rational number in its lowest terms.)

(ii) Can there be an infinite geometric series with a limiting sum of $\frac{5}{8}$ and a first term of 2? (All working and reasoning must be shown.)

(iii) If $x^2 = (2a-x)(2b-x)$ show that $\frac{1}{a}, \frac{1}{x}, \frac{1}{b}$ are in arithmetic sequence.

QUESTION 4

(i) Differentiate with respect to x:

(a) $\frac{1}{x^2}+\sqrt{x}$ (b) $3\sin x + 4\cos x$ (c) $(3x+2)^5$

(ii) Two particles A and B move along a straight line so that their displacements from the origin at time t are given respectively by $s_A = 12t+5$ and $s_B = 6t^2 - t^3$

(a) Which is moving faster when $t = 1$?

(b) When do the particles travel at the same speed?

(c) What is the acceleration of the particle B when $t = 3$?

(d) What is the maximum positive displacement of the particle B?

QUESTION 5

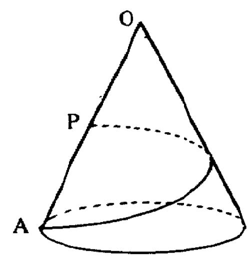
A thin sheet of smooth metal is in the shape of a sector of a circle with OA, OB as bounding radii each of length 10 cm, and the angle AOB is 60°.

(i) Find the length of the arc AB and the area of the sector.

(ii) The sheet is now bent to form a right circular cone by welding the bounding radii OA, OB together (and inserting a circular disc to close in the cone at the base.)

(a) Find the volume of this cone.

(Note: The volume of a right circular cone is, in the usual notation, $\frac{1}{3}\pi r^2 h$.)



(b) On the surface of this cone a thin string is pulled tight starting with one end fixed at the point A and passing once round the cone to the other end P which is at the midpoint of OA (as illustrated in the diagram). Find the length of this string.

QUESTION 6

(i) A(1, 8), B(3, 7) and C(-2, 5) are three vertices of a parallelogram ABCD. Find the coordinates of D.

(ii) A point P(x, y) moves so that its distance from the y-axis is always half its distance from the line $y = 1$. Find the equation of the locus of P.

(iii) The curves $y = \log_e x$ and $y = \frac{e}{x}$ intersect at the point $(e, 1)$. Show that the acute angle θ between the two curves at this point is given by $\tan\theta = \frac{2e}{e^2-1}$.

QUESTION 7

(i) State (a) the natural (largest possible) domain; and

(b) the range of the function f for which $f(x) = \sqrt{1-x^2}$

(ii) Sketch the graph of the function in (i) (not on graph paper).

(iii) Without sketching, determine whether the following curves cross the x-axis or not:

(a) $y = 3x^2 - 4x + 5$ (b) $y = 2x^2 + 3x - 4$

(iv) For the curve $y = x^3 - 3x^2 - 12$ find the stationary points and determine their nature. Also find any points of inflexion.

QUESTION 8

(i) Evaluate (a) $\int_1^2 (x^2 + \frac{1}{x^3})\,dx$ (b) $\int_0^1 \frac{dx}{4+3x}$

(ii) (a) Indicate, by shading on a diagram, the region in the positive quadrant bounded by the y-axis, the line $y = 2$ and the curve $y = e^x$. Calculate the area of this region.

(b) Use Simpson's Rule, with three function values, to calculate the volume of solid generated when the region in (a) is rotated about the y-axis.

QUESTION 9

(i) An urn contains 4 black and 3 white balls. Two balls are drawn at random and placed in a hat. What is the probability that the hat contains

(a) two white balls (b) a white and a black ball?

(ii) In a certain strain of plant the probability that a seed will produce a pink flower is $\frac{1}{4}$. Determine the least number of seeds that must be planted in order that the probability of obtaining at least one pink flower exceeds 0.99.

QUESTION 10

(i) Two cars, represented by points A and B, are travelling due east and due north respectively along two roads represented by two straight lines intersecting at O. At a certain instant, car A is 2 kilometres west of O and car B is 1 kilometre south of O, the former travelling at a constant speed of 1 kilometre per minute and the latter at constant speed V kilometres per minute.

(a) What value of V will cause a collision?

¶ (b) Prove that, in the course of motion, the minimum distance between the cars is $\frac{|2V-1|}{\sqrt{V^2+1}}$ km.

¶ (ii) Determine the range of values of V which cause collision if the above problem is made more realistic by representing the cars, not by points A and B, but by rectangles, each of length a kilometres and width b kilometres, with their centres at A and B.