N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1975
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (12 Marks)

(i) Indicate by shading the region in which the following relation holds:

$9 < x^2 + y^2 < 25$ (Do not use graph paper)

(ii) Find, correct to 4 decimal places (a) tan 160° (b) cosec 250°

(iii) Draw separate sketches (showing main features - not on graph paper) of:

(a) $xy = 1$ (b) $y = |x|$ (c) $y = x^2 - 4$

(iv) Find the point that divides the interval joining (-2, 3) and (3, -7) internally in the ratio 3:2.

QUESTION 2 (9 Marks)

(i) Show that the points (2, 7), (5, 13) and (-4, -5) are collinear.

(ii) Show that the line $3x + 4y + 10 = 0$ is a tangent to the circle with centre at the origin and radius 2 units.

(iii) The points A, B and C are equally spaced on the circumference of a circle radius a. Find the area of the triangle ABC.

QUESTION 3 (9 Marks)

(i) Differentiate (a) $x^3 + 4x$ (b) $x\log_e x$

(ii) Write down a primitive function (i.e. an indefinite integral) of

(a) $x^3 + \sqrt{x}$ (b) $\frac{1}{x+3}$ (c) $\frac{1}{\sqrt{x}}$

(iii) For the curve $y = ax^2 + bx + c$, where a, b, c are constants, it is given that $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 1$. Show that $a = c$.

QUESTION 4 (9 Marks)

(i) The sum of the first three terms of an arithmetic progression is 27 and the sum of the next three terms is 63. Find the first term and the common difference of this progression.

(ii) Express the following in the simplest possible form, without the use of negative indices:

(a) $\frac{x^{-1}+y^{-1}}{x+y}$ (b) $(8x^6)^{1/3} . x^{-3}$

(iii) Find the first and second derivatives of $\frac{e^x}{x}$.

QUESTION 5 (10 Marks)

(i) For the two acute angles A and B it is given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$; find the exact value of:

(a) $\cos A$ (b) $\sin B$ (c) $\sin(A + B)$

(ii) Two artillery guns are situated 3 kilometres apart at positions X and Y respectively. They are both aiming at a target T. The angles XYT and YXT are respectively 72° and 78°. Find the distance between the target and the gun nearer to it.

QUESTION 6 (10 Marks)

(i) Find all roots of the equation $x^4 - 13x^2 + 36 = 0$

(ii) Find the values of k for which the quadratic equation $x^2 - (k+3)x + (k+6) = 0$ has real roots.

(iii) A man has four pairs of socks, each pair a different style. If he selects two socks at random, what is the probability that they form a matching pair?

QUESTION 7 (10 Marks)

(i) Find the volume enclosed by the surface generated when the curve $x^2 + 4y^2 = 16$ is rotated about the x-axis.

(ii) Sketch, but not on graph paper, the graph of $y = 2\cos 3x$ for $-\pi \le x \le \pi$. What is the period of the function $2\cos 3x$?

QUESTION 8 (10 Marks)

(i) Find the stationary points of the curve $y = x^3 - 7x^2 - 5x$ and establish their nature by the use of calculus.

(ii) The two perpendicular lines $3x + 2y = 12$, $2x + ay = b$ intersect at the point (2, 3). Find the values of a and b.

QUESTION 9 (10 Marks)

(i) The position of a particle on a line at time t is given by $x = 3 + e^{2t}$, where x is its distance from a fixed point O. Find:

(a) its initial position and initial velocity;

(b) an expression for its acceleration in terms of t;

(c) an expression for its acceleration in terms of x.

(ii) The velocity, v metres per second, of a particle at time t seconds is given by $v = t^3 - 3t^2 + 4t - 6$. Show that the acceleration is never less than 1 metre per second per second.

QUESTION 10 (10 Marks) **{Suitable for 3 Unit Students}**

P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$ and the chord PQ intersects the axis of the parabola on the directrix.

(i) Find the equation of the chord PQ.

(ii) Find the relation between p and q.

(iii) Find the coordinates of M, the midpoint of the chord PQ in terms of a, p and q.

(iv) Show that M lies on the parabola $x^2 = 2a(y + a)$.

ALTERNATIVE QUESTION 10 **{Suitable for 2 Unit Students - based on above}**

P(8, 8) and Q$(2, \frac{1}{2})$ lie on the parabola $x^2 = 8y$.

(i) Find the equation of the chord PQ.

(ii) Show that the chord PQ intersects the axis of the parabola on the directrix.

(iii) Find the coordinates of M, the midpoint of the chord PQ.

(iv) Show that M lies on the parabola $x^2 = 4(y+2)$.

(v) The line through M parallel to the axis of the parabola meets the parabola at T. Prove that the tangent at T is parallel to the chord PQ.

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