NEW SOUTH WALES

Higher School Certificate

Mathematics Extension 2

Exercise 2/67

by James Coroneos*

1. Express the following in the form $a+ib$, where a, b are real.

(i) $(3+2i)+(2-3i)$ **(ii)** $(3+2i)-(2-3i)$
(iii) $(3+2i)(2-3i)$ **(iv)** $\frac{3+2i}{2-3i}$
(v) $(4-6i)+(3+2i)$ **(vi)** $(3+2i)-(7+4i)$
(vii) $(-4+7i)(3-2i)$ **(viii)** $\frac{6-i}{1+3i}$
(ix) $(2+i)^2$ **(x)** $(2+i)^{-2}$

2. Simplify

(i) $(-3+3i)^2$ **(ii)** $(-3+3i)^4$
(iii) $i^5+3i^4-5i^3+6i^2-3i$ **(iv)** $(2i-3)(3i^2-5i-7)$
(v) $(2\sqrt{3}i+3\sqrt{2}i)(4\sqrt{3}i-5\sqrt{2}i)$ **(vi)** $(3\sqrt{-7}-5\sqrt{-2})(3\sqrt{-7}+5\sqrt{-2})$
(vii) $(e^i+e^{-i})(e^i-e^{-i})$ **(viii)** $\frac{2-\sqrt{-3}}{2+\sqrt{-3}}$
(ix) $\frac{3\sqrt{-2}+2\sqrt{-5}}{3\sqrt{-2}-2\sqrt{-5}}$ **(x)** $\frac{1+i}{1-i}$

3. Simplify

(i) $\frac{(1+i)^2}{3-i}$ **(ii)** $\frac{3+2i}{2-5i}+\frac{3-2i}{2+5i}$
(iii) $\frac{a+ix}{a-ix}-\frac{a-ix}{a+ix}$ **(iv)** $\frac{1}{(2+i)^2}-\frac{1}{(2-i)^2}$
(v) $\{\sqrt{9+40i}+\sqrt{9-40i}\}^2$ **(vi)** $\frac{(x+i)^3-(x-i)^3}{(x+i)^2-(x-i)^2}$
(vii) $(2+i)(3+i)(4+i)$ **(viii)** $\frac{3+4i}{2-i}+\frac{5-2i}{3+i}$
(ix) $3(2-3i)(3+4i)-(5-i)^2$ **(x)** $\frac{(2+i)(3-2i)}{(2+3i)(1+i)}$

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4. Form the equation whose roots are

(i) $2-\sqrt{3}, 2+\sqrt{3}$ **(ii)** $2i, 4i$ **(iii)** $2-3i, 2+3i$ **(iv)** $i, 1-i$ **(v)** $a+ib, a-ib$

5. **(a)** Find the value of x^3+x^2-x+22 when $x=1+2i$.
(b) Show that $x=i$ is a root of the equation $x^3+(1-i)x^2+(1-2i)x=1+i$.
(c) If $x=1+i$ is a root of $x^3+ax+4=0$, show that $a=-2$.

6. If $u=3-4i$, $v=4-3i$ find
(i) $u+iv$ **(ii)** uv **(iii)** $\frac{u}{v}$ **(iv)** u^2+v^2

7. Given that $z=x+iy$, express in the form $X+iY$
(i) z^2 **(ii)** $\frac{1}{z}$ **(iii)** $\frac{z-1}{z+1}$ **(iv)** z^{-2}

8. Show that i^n, where n is integral, has one of four values $\pm 1, \pm i$.

9. Solve for w_1, w_2 : $\begin{cases} 2w_1+3iw_2 & =0 \\ (1-i)w_1+2w_2 & =i-7 \end{cases}$

10. If $z=\cos\theta+i\sin\theta$, prove that $\frac{2}{1+z} = \frac{1+\cos\theta-i\sin\theta}{1+\cos\theta} = \frac{2\cos^2\frac{\theta}{2}-2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$
$= 1-i\tan\frac{\theta}{2}$

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