Fitzpatrick's 4 Unit Specimen Papers

PAPER 1

1. **(i)** Prove that: **(a)** $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\mathrm{d}x}{1-x^2} = \log_e 3$;

(b) $\int_0^{\frac{\pi}{4}} \tan\theta \, \mathrm{d}\theta = \frac{1}{2}\log_e 2$.

(ii) **(a)** If $t = \tan\frac{x}{2}$, show that $\cos x = \frac{1-t^2}{1+t^2}$.

(b) Hence, show that $\int \sec \mathrm{d}x = \log_e\left(\tan\frac{x}{2} + \frac{\pi}{4}\right) + c$.

(iii) If a curve with equation $y = \mathrm{f}(x), a \le x \le b$, is rotated about the x-axis, it will generate a surface in space. It can be shown that the area of this surface is given by the integral $2\pi\int_a^b y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$. Use this result to show that the surface of a sphere of radius r is $4\pi r$.

2. **(i)** **(a)** Find the cartesian equation of the curve whose parameric equations are $x = 2\sec\theta, y = \tan\theta$.

(b) Find the equation of the normal to this curve at the point P, where $\theta = \frac{\pi}{4}$.

(c) Show that this normal intersects the curve again at a point Q whose y-coordinate is $-\frac{17}{7}$.

(d) Find the volume of the solid of revolution formed by rotating about the y-axis the region enclosed by the curve and the line segment PQ. (Use the method of cylindrical shells.)

(ii) **(a)** Prove that the equation of the tangent to the conic $ax^2 + by^2 = 1$ at the point (x_1, y_1) is $ax_1x + by_1y = 1$.

(b) Hence prove that, if the line $px + qy + r = 0$ is a tangent to the conic $ax^2 + by^2 = 1$, then $\frac{p^2}{a} + \frac{q^2}{b} = r^2$.

3. **(i)** **(a)** Express $1 + i$ and $1 - i$ in the form $r(\cos\theta + i\sin\theta)$.

(b) Hence use De Moivre's theorem to find $(i+i)^9 + (1-i)^9$.

(ii) If z is one of the fifth roots of unity, not 1, show that:

(a) $1 + z + z^2 + z^3 + z^4 = 0$;

(b) $z - z^2 + z^3 - z^4 = 2\left(\sin\frac{2\pi}{5} - \sin\frac{\pi}{5}\right)i$.

(iii) Draw a neat sketch of the region of the complex plane specified by $|z - (2\sqrt{2} + i2\sqrt{2})| \le 2$ and hence find:

(a) the maximum and minimum values of $|z|$;

(b) the maximum and minimum values of Arg z.

4. (i) Two circles intersect at A and B. The line PAQ through A cuts one circle at P and the other at Q. AB is produced to a point C and the lines CP and CQ cut the circles at D and E respectively. Prove that:

(a) triangles CBD and CAP are similar;

(b) the points D, B, E and C are concyclic;

(c) $\frac{CD}{CE} = \frac{CQ}{CP}$.

(ii) $A(-a, 0)$ and $B(a, 0)$ are two fixed points in the cartesian plane and $P(x, y)$ is such that $\angle APB = \theta$.

(a) If the gradients of PB and PA are m_1 and m_2 respectively show that $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$.

(b) Find the equation of the locus of P if $\angle APB = 60^\circ$.

(c) Show that the locus is an arc of a circle and state its centre and radius.

(d) Give a geometrical reason why you would expect the locus to be part of a circle.

5. (i) Find the number of ways of arranging m white cubes and n black cubes in a row, $m > n$, if no two black cubes are together and if there is a white cube at each end.

(ii) Two friends, Alex and Joyce, plan to meet at a particular place between 1 p.m. and 2 p.m. They decide that, upon arrival, each would wait for 10 minutes only and then leave if the other had not turned up. Show that the probability that they will meet is $\frac{11}{36}$.

6. (i) One end of a light inextensible string, of length 1m, is fixed at a point O. A mass of 1kg, attached to the other end moves in a horizontal circle whose centre is vertically below O. If the tension in the string is equal to the weight of a 2kg mass find:

(a) the angle which the string makes with the vertical;

(b) the speed of the moving 1kg mass, correct to two significant figures. (Take acceleration due to gravity as 9.8 m/s^2.)

(ii) Find the angle at which a road must be banked so that a car may round a curve of radius 150 m at 180 km/h without sliding even if the road is smooth.

7. (i) A toy rocket is projected vertically upwards and it rises with an acceleration of $(9 - 5t)g$ m/s^2 for the first two seconds and thereafter freely against gravity. Find:

(a) the maximum speed;

(b) the maximum height reached.

(ii) If T seconds is the time taken for a body projected vertically upwards to reach a height of h metres, and T' seconds is the time taken to reach the ground again from this point, show that:

(a) $h = \frac{1}{2}gTT'$;

(b) the maximum height reached is $\frac{1}{2}g(T+T')^2$. (Neglect air resistance.)

8. For the graph of the function defined by $\mathrm{f}(x) = \sin^{-1}\frac{2x}{1+x^2}$:

(i) state the largest possible domain and range;

(ii) write down the derivative of $\sin^{-1}\frac{2x}{1+x^2}$;

(iii) show that the derivative is defined for all values of x except $x = \pm 1$;

(iv) determine what the gradient of the curve is as $x \to \pm 1$;

(v) show that the graph has no turning points;

(vi) show that f is an odd function;

(vii) show that there is a point of inflexion at the origin;

(viii) sketch the graph of the function.

PAPER 2

1. (i) Find the exact value of:

(a) $\int_1^3 \frac{e^3}{1+2e^x}\,\mathrm{d}x$;

(b) $\int_0^2 \frac{1}{x^2-2x+2}\,\mathrm{d}x$.

(ii) Find real numbers A, B and C such that $\frac{2x^2+19x-36}{(x+3)(x-2)^2} = \frac{A}{x+3} + \frac{B}{(x-2)^2} + \frac{C}{x-2}$.
Hence show that $\int_3^5 \frac{2x^2+19x-36}{(x+3)(x-2)^2}\,\mathrm{d}x = 8\log_e 3 - 6\log_e 2 + \frac{4}{3}$.

(iii) Write down the derivative of $x^m e^x$ and hence deduce the value of:

(a) $\int_1^2 xe^x\,\mathrm{d}x$; (b) $\int_1^2 x^2e^x\,\mathrm{d}x$.

2. (i) Draw neat sketch graphs of:

(a) $y = (x-1)(x+1)$;

(b) $y = \frac{1}{(x-1)(x+1)}$;

(c) $y = \sqrt{(x-1)(x+1)}$;

(d) $y^2 = (x-1)(x+1)$.

(ii) Find a general solution to the equation $\cos\theta + \cos 2\theta + \cos 3\theta + \cos 4\theta = 0$.

3. **(i)** Given $z = \sqrt{3} + i$ and $w = 1 - i$ express each of the following in the form $a + ib$ where a and b are real numbers.

(a) z^6; **(b)** $(\overline{z})^3$; **(c)** $\frac{z}{w}$; **(d)** $\left|\frac{z}{w}\right|$.

(ii) If $z = x + iy$, find the cartesian equation of the line given by $(1+i)z + (1-i)\overline{z} = -2$. Sketch on the Argand diagram that part of the line for which $\arg z \leq \frac{\pi}{2}$.

(iii) Sketch the regions of the Argand plane defined by:

(a) $|z - i| \leq 1$; **(b)** $\Im(z) \leq 0$ and $\Re(z) \geq 2$.

Hence find **(c)** the shortest distance between the two regions.

4. **(i)** A metal ball of mass m is falling vertically in a tank of oil under constant gravity. If the resistance to the motion is $mkv (k > 0)$ when the speed is v, find an expression for the time t taken for the ball to acquire a speed v from rest. Transpose this expression to express v in terms of t and sketch the graph of v against t.

(ii) A particle of mass $g/2$ kg is moving along the x-axis under the influence of a force whose magnitude is $(3 - 2x)$ kg wt. If the speed is 4 m/s when $x = 0$, show that the motion is simple harmonic and find the centre of motion, the amplitude and the frequency.

5. **(i)** $P\left(ct, \frac{c}{t}\right)$ and $Q\left(\frac{c}{t}, ct\right)$ are two point on the rectangular hyperbola $xy = c^2$. R and S are two other points on the curve such that P, Q, R and S are the vertices of a rectangle.

(a) Find the coordinates of R and S in terms of t.

(b) Prove that it is impossible for these four points to be vertices of a square.

(ii) Let $P(a\cos\theta, b\sin\theta)$ where $0 < \theta < \frac{\pi}{2}$ is a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The tangent at P meets the x-axis at A and the y-axis B.

(a) Write down the equation of the tangent at P.

(b) Find the coordinates of A and B.

(c) Show that the minimum length of the line segment AB is $a + b$ units.

(d) Find, in terms of a and b, the coordinates of P when the length of AB is a minimum.

6. **(i)** Consider 12 points in a plane, no three of which are in a straight line except five of them which are in the same straight line. By joining the points find:

(a) the number of straight lines;

(b) the number of triangles which can be formed.

(ii) If $x + y + z = 1$ show that:

(a) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq 9$;

(b) $(1 - x)(1 - y)(1 - z) > 8xyz$.

7. (i) If S is the sum of n terms of a geometric sequence, P is the product of these terms and R is the sum of the reciprocals of these terms, prove that $\left(\frac{S}{R}\right)^n = P^2$.

(ii) A vertical tower of height h m stands at the top of a slope of inclination α to the horizontal. An observer at the foot of the slope finds the angle of elevation of the top of the tower to be β. The observer walks a distance d m up a line of greatest slope, towards the foot of the tower, and finds the angle of elevation of the top of the tower to be γ. If the observer is then x m away from the foot of the tower, prove that:

(a) $h = \frac{d\sin(\beta-\alpha)\sin(\gamma-\alpha)}{\sin(\gamma-\beta)\cos\alpha}$;

(b) $x = \frac{d\sin(\beta-\alpha)\cos\gamma}{\sin(\gamma-\beta)\cos\alpha}$.

8. (i) The base of a solid is the circle $x^2 + y^2 = 4$ and every section of the solid by a plane perpendicular to the x-axis is:

(a) a square;

(b) an equilateral triangle;

(c) a right angled isosceles triangle with its hypotenuse in the plane of the circle. Find, in each case, the volume of the solid.

(ii) Prove that, for a fixed value of r and for $n \geq 2$, ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$:

(a) using the definition of ${}^nC_r$;

(b) using the fact that ${}^nC_r = \frac{n!}{r!(n-r)!}$;

(c) by the method of induction.

PAPER 3

1. (i) Find the exact value of:

(a) $\int_0^1 \frac{x}{\sqrt{9-x^2}}\,\mathrm{d}x$; **(b)** $\int_0^1 \frac{1}{\sqrt{9-x^2}}\,\mathrm{d}x$; **(c)** $\int_0^1 xe^{-x}\,\mathrm{d}x$.

(ii) Express $\frac{12}{x^3+8}$ in partial fractions and hence show that
$\int \frac{12}{x^3+8}\,\mathrm{d}x = \log_e \frac{x+2}{\sqrt{x^2-2x+4}} + \sqrt{3}\tan^{-1}\frac{x-1}{\sqrt{3}}$.

2. **(i)** Write down the derivative of $e^{-x}\sin x$.

(ii) Show that the derivative of $y = e^{-x}\sin x, x \geq 0$ crosses the x-axis at $x = 0, \pi, 2\pi, \ldots$ and has turning points at $x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \ldots$.

(iii) Draw a rough sketch of the graph of $y = e^{-x}\sin x, x \geq 0$.

(iv) Calculate, by integration, the area of each of the first three regions bounded by the curve and the x-axis.

(v) Show that the magnitudes of the areas of all such regions form the terms of an infinite geometric sequence with common ratio $e^{-\pi}$.

3. **(i)** If $z = -1 + i\sqrt{3}$, write down the value of:

(a) $\overline{z}$; **(b)** $|z|$; **(c)** z^4; **(d)** $\arg(z^4)$.

(ii) Verify that $z = -1+i\sqrt{3}$ is a root of the equation $z^4 - 4z^2 - 16z - 16 = 0$ and hence find the other roots.

(iii) **(a)** Sketch the region of the Argand diagram consisting of the set of all values of z for which $1 \leq |z| \leq 3$ and $-\frac{\pi}{6} \leq \arg z \leq \frac{\pi}{6}$.

(b) Show that the locus specified by $|z+5i| - |z-5i| = 6$ is a hyperbola. Write down its cartesian equation and its asymptotes.

4. **(i)** P is any point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$. N is the foot of the perpendicular from P to the x-axis. NP produced meets the hyperbola $x^2 - y^2 = 9$ at Q. If the normals at P and Q to the respective hyperbolas meet at R, find the equation of the locus of R.

(ii) ABC is an acute angled isosceles triangle with $AB = AC$. AB is produced to O such that AC is a tangent to a circle, centre O. This circle meets CB produced at D. Prove that angle DOA is a right angle.

5. **(i)** A gun is so aimed that the shell it fires strikes a target released simultaneously from an aeroplane flying horizontally towards the gun at a speed V at height h. If the aeroplane was a horizontal distance x from the gun when the target was released, and the shell strikes the target at half this distance, show that the gun was aimed at a point h vertically above the aeroplane at the instant of release, and that the shell left the gun with speed $\frac{V}{x}\sqrt{x^2 + 4h^2}$.

(ii) A particle of mass m is projected vertically upwards with speed U. When the speed is v, the air resistance is kmv^2. If the particle returns to the point of projection with speed V, show that $\frac{1}{V^2} = \frac{1}{U^2} + \frac{k}{g}$.

6. **(i)** Five girls and three boys arrange themselves at random in a straight line. How many different arrangements are possible if:

(a) no two boys are to be together;

(b) the girls are to be together.

What is the probability of these arrangements?

(ii) If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, find, in terms of p, q and r, the values of:

(a) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$;

(b) $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$.

(iii) Find all x such that $\tan 3x = \cot 2x$.

7. **(i)** Prove, by induction, that the maximum number of pieces into which a pizza can be sliced by n straight cuts from edge to edge is $\frac{1}{2}(n^2+n+2)$.

(ii) A particle of mass 1 kg on a smooth table is fastened to one end of a light inextensible string which passes through a small hole in the table and supports a particle of mass 2 kg at the other end. The 1 kg mass is held 0.5 m from the hole and then projected with a speed of v m/ so that it describes a circle of radius 0.5 m. Find the value of v.

8. **(i)** With the aid of De Moivre's theorem and the binomial theorem, express:

(a) $\cos 3\theta$ in terms of $\cos\theta$;

(b) $\sin 3\theta$ in terms of $\sin\theta$;

(c) $\tan 3\theta$ in terms of $\tan\theta$ from **(a)** and **(b)**.

(ii) A die is loaded so that the probability of throwing a one is $\frac{p}{3}$ and of throwing a six is $\frac{1-p}{3}$ and of throwing a two, three, four or five is each the same. What value can p have to maximise the probability of a score of seven as a result of throwing the die twice?

PAPER 4

1. **(i)** Evaluate:

(a) $\int_0^{\frac{1}{2}} \sqrt{1-x^2}\, \mathrm{d}x$;

(b) $\int_0^{\frac{1}{2}} \frac{2-x^2}{\sqrt{1-x^2}}\, \mathrm{d}x$.

(ii) Use the substitution $x = \cos 2\theta$ to evaluate $\int_{\frac{1}{2}}^{1} \sqrt{\frac{1-x}{1+x}}$.

(iii) Show, using derivatives, that, if $x > 0, \log_e(1+x) > x - \frac{1}{2}x^2$.

2. **(i)** If $z = \sqrt{2} + i\sqrt{2}$, express each of the following in modulus-argument form:

(a) $z\overline{z}$; **(b)** $\frac{1}{z}$; **(c)** z^5.

(ii) Find complex numbers z and w so that $z + iw = 1$ and $iz + w = 1 + i$ are both satisfied.

(iii) Show that the locus specifed by:

(a) $|z - (1 - i)| = |z - (4 + 4i)|$ is a straight line;

(b) $2|z - (1 + i)| = |z - (4 + 4i)|$ is a circle and find the equation in each case.

3. (i) $P(a\cos\theta, b\sin\theta)$ and $Q(a\cos\phi, b\sin\phi)$ are variable points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If $\theta + \phi = \frac{\pi}{2}$:

(a) express the coordinates of Q in terms of θ;

(b) find the coordinates of the midpoint M of the chord PQ in terms of θ;

(c) show that the locus of M is a straight line through the origin.

(ii) Find, by integration, the volume of the solid generated by rotating the circle $x^2 + y^2 = a^2$ about the line $x = b(b > a)$.

4. (i) A car of mass m moves with constant speed in a horizontal circle of radius r on a banked race track, which is banked at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The frictional force between the tyres and the track varies according to the speed from a limit of $\frac{5}{11}\ mg$ down the track to mg up the track. Find in terms of r and g, the range of speeds at which the car can negotiate this track without the tyres slipping on the road surface.

(ii) An explosion at a point on level ground hurls debris in all directions with speed 14 m/s. Prove that a person distant $10\sqrt{3}$ m horizontally from the explosion could be struck by debris at two instants $\frac{10}{7}(\sqrt{3} - 1)$ seconds apart. (Neglect air resistance and use $g = 9.8$.)

5. (i) A motorist travelling at a speed of 72 km/h reaches a level section of road and puts the car in neutral during which time the retardation is proportional to the speed. The motorist observes that after travelling 80 m along this section, the speed has fallen to 36 km/h. How long did this take?

(ii) A smooth hollow cone of semi-vertex angle α is placed with its axis vertical and vertex downward. A particle of mass m kg moves in a horizontal circle on its inside surface, making n revolutions per second. Find an expression for the distance of the particle from the axis of the cone.

6. (i) For the curve with equation $y = \frac{x^3 - 3x}{(x-1)^2}$, determine the x-coordinate of, and the nature of, the stationary point. Let A be the region bounded by the curve and the lines with equations $y = x + 2, x = 3$ and $x = b$

where $b > 3$. Show that for all values of b, the area of A cannot exceed the value 1. For what values of b will the difference between 1 and the area of A be less than 10^{-3}?

(ii) Find the values which the real numbers a and b must take for $z = 1$ to be a root of the equation $iz^2 + (ia - 1)z + (i - b) = 0$. For these values of a and b write down expressions for the sum and product of the roots of this equation and hence obtain the second root of the equation in $a + ib$ form.

7. (i) $P\big(ct, \frac{c}{t}\big)$ is a variable point on the hyperbola $xy = c^2$. The tangent at P cuts the x-axis and the y-axis at A and B respectively. Q is the fourth vertex of the rectangle with vertices A, B and the origin. Show that Q lies on the hyperbola $xy = 4c^2$.

(ii) A cylindrical hole through the centre of a sphere is 6 cm long. Fing the remaining volume of the sphere.

8. (i)(a) Prove, by induction, that $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \cdots + {}^nC_nx^n$.

(b) By considering the coefficient of x^n in the expansion of $(1+x)^n(1+x)^n$ in two different ways, prove that:

(α) $({}^nC_0)^2 + ({}^nC_1)^2 + ({}^nC_2)^2 + \cdots + ({}^nC_n)^2 = \frac{(2n)!}{(n!)^2}$;

(β) ${}^nC_0{}^nC_1 + {}^nC_1{}^nC_2 + \cdots + {}^nC_{n-1}{}^nC_n = \frac{(2n)!}{(n+1)!(n-1)!}$.

(ii) ABC is a triangle and a transversal cuts AB at R, BC at P and CA produced at Q. A line through A parallel to BC cuts the transversal at K. Name two pairs of similar triangles and prove that $\frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA} = 1$.

PAPER 5

1. (i) Evaluate:

(a) $\int_0^1 \frac{1+x}{1+x^2}\ dx$; **(b)** $\int_1^2 x \log_e x\ dx$.

(ii) The region bounded by the circle $x^2 + y^2 = 1$ and the parabola $2y^2 = 3x$ is rotated about the x-axis. Find the volume of the solid of revolution.

2. (i) If $z = a + ib, b \neq 0$ simplify:

(a) $\left|\frac{\bar{z}}{z}\right|$; **(b)** $\frac{i(\Re(z) - z)}{\Im(z)}$; **(c)** $\arg z + \arg\left(\frac{1}{z}\right)$; **(d)** $\frac{z\bar{z}}{|z|^2}$.

(ii) Use De Moivre's theorem to find the values of n for which $(\sqrt{3} + i)^n - (\sqrt{3} - i)^n = 0$ where n is a positive integer.

(iii) Draw a neat sketch of the locus defined by $|z|^2 - 2iz + 2t(1 + i) = 0$ where $z = x + iy$ and x, y and t are real numbers. For what values of t

can x and y be found so that z satisfies the given equation?

3. **(i)** If $f(x) = \cos^2 x + \cos^2\left(x + \frac{2\pi}{3}\right) + \cos^2\left(x - \frac{2\pi}{3}\right)$, find $f'(x)$ and prove that it is zero for all real x. Hence show that $f(x) = \frac{3}{2}$ for all real x.

(ii) If $x > 0, n$ a positive integer > 1 and $nx < 1$, show that $1 + nx < (1+x)^n < \frac{1}{1-nx}$.

4. **(i)** For the function f where $f(x) = \sqrt{\frac{1+x}{1-x}}, 1 \leq x \leq 1$, obtain the inverse function g, specifying its domain and range. Sketch the graph of f and g on one set of axes.

(ii) $P(x_1, y_1)$ is any point on the rectangular hyperbola $x^2 - y^2 = a^2$ whose foci are at S and S'. Prove that:

(a) $PS.PS' = PO^2$ where O is the origin;

(b) if the normal at P meets the axes at Q and R, then $PQ{=}PR{=}PO$.

5. **(i)** Show that $2\cos(n-1)\theta\cos\theta - \cos(n-2)\theta = \cos n\theta$. Hence express $\cos 5\theta$ as a polynomial in $\cos\theta$.

(ii) By considering the roots of the equation $16x^4 - 20x^2 + 5 = 0$, show that:

(a) $\cos\frac{\pi}{10}\cos\frac{3\pi}{10}\cos\frac{7\pi}{10}\cos\frac{9\pi}{10} = \frac{5}{16}$;

(b) $\sin\frac{\pi}{5}\sin\frac{2\pi}{5} = \frac{\sqrt{5}}{4}$.

6. **(i)** A particle of mass m, suspended by an inelastic wire from a point O, moves with constant speed V in a horizontal circle in such a way that the wire makes a constant acute θ with the downward vertical through O. Make a sketch showing the forces acting on the particle and find the length of the wire in terms of V, θ and g.

(ii) A particle is brought to top speed with an acceleration that varies linearly with the distance travelled. It starts from rest with an acceleration of 3 m/s^2 and reaches top speed in a distance of 160 m. Find:

(a) the top speed;

(b) the speed when the particle has moved 80 m.

7. **(i)** How many different selections of four letters may be made from the letters of the word WEDNESDAYS and in how many ways may they then be arranged in a row?

(ii) Three students work independently to solve a certain mathematics problem. Their respective probabilities of solving it are 0.8, 0.7, 0.6. What is the probability that at least two of them solve it?

8. (i) $P(3\cos\theta, 2\sin\theta)$ and $Q(3\cos\phi, 2\sin\phi)$ are variable points on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. The chord PQ subtends a right angle at the point $(3, 0)$. Show that PQ passes through a fixed point on the major axis.

(ii) A particle of weight 4 newton, initially at rest, falls vertically in a resisting medium with a retarding force of kv newton, where k is a constant and v m/s is the velocity of the particle at any time t seconds after release.

(a) Evaluate k if the particle eventually attains a constant speed of 20 m/s.

(b) Find the velocity of the particle at any time t.

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