NEW SOUTH WALES

Higher School Certificate

Mathematics Extension

Exercise 11/67

by James Coroneos*

- **1.** If P represents the complex number z, sketch the locus of P if
 - (i) |z| = 4 (ii) $|z| \le 4$ (iii) $|z 1| \le 3$ (iv) |z + 3i| < 1 (v) |2z 3| = 1
 - (vi) |z 1 2i| = 4 (vii) $\arg z = \pi/2$ (viii) $\arg z = -\pi/3$ (ix) $\Re(z) = 2$
 - (x) $\Im(z) = -2$ (xi) 1 < |z| < 2 (xii) $3 < |z| \le 4$ (xiii) $2 \le |z| \le 5$
 - (xiv) $1 \le |z+2| \le 2$ (xv) $2 \le \Im(z) < 3$ (xvi) $2 < \Re(z) \le 3$
 - (xvii) $0 < \arg z < \pi/6$ (xviii) $\pi/2 \le \arg z \le 2\pi/3$ (xix) $|\frac{1}{z}| \ge \frac{1}{9}$
 - (xx) 1 < |z 1 + i| < 2 (xxi) $\Re(z^2) = 0$ (xxii) $\Im(z^2) = 2$
 - (**xxiii**) $\Re(z) = |z 1|$ (**xxiv**) $0 < \Re(iz) \le 2$ (**xxv**) $\Re(z iz) \ge 2$.

2. Mark in *clearly* on the Argand diagram, the regions of the *z* plane satisfied by: (i) $\Re(z) \ge 1$ and $1 \le \Im(z) \le 2$ (ii) $\Re(z) \ge 1$ or $1 \le \Im(z) \le 2$ (iii) 3 < |z| < 4 and $\pi/3 < \arg z < 2\pi/3$ (iv) 3 < |z| < 4 or $\pi/3 < \arg z < 2\pi/3$ (v) $|z| \le 3$ and $\pi/4 < \arg z \le \pi$ (vi) |z| < 4 or $\pi/4 \le \arg z < 3\pi/4$ (vii) $2 < |z| \le 3$ and $\Im(z) > 1$ (viii) $-1 \le \Re(z) \le 1$ or $\Im(z) \le 3$ (ix) $\Im(z) \ge 1$ and $0 \le \arg z \le \pi/4$ (x) $\Im(z) < 2$ or $\pi/3 \le \arg z \le 2\pi/3$ (xi) $1 \le \Re(z) \le 2$ and $2 \le \Im(z) \le 3$ (xii) $\Re(z) < 2$ or $-\pi/3 < \arg z \le \pi/3$ (xiii) 1 < |z+i| < 2 and $\pi < \arg z < 3\pi/2$ (xiv) |z+i| = 1 or $0 \le \Re(z) \le 1$ (xv) $-4 \le \Im(z) \le 4$ and $|z| \ge 5$ (xvi) $-4 \le \Im(z) \le 4$ or $|z| \ge 5$ (xvii) |2z - 3| < 2 and $\pi/6 < \arg z < \pi/2$ (xviii) |z| > 2 or |z| < 1

3. Consider the roots of the quadratic equation $z^2 + az + 9 = 0$. If z_1 and z_2 are the roots of this equation and 'a' is real, draw the locus traced out by the two roots in the complex plane as a takes on all real values. [Hint: consider $a^2 \ge 36; a^2 < 36$]



^{*}Other resources by James Coroneos are available. Write to P.O. Box 25, Rose Bay, NSW, 2029, Australia, for a catalogue. TYPESET BY A_MS -T_EX.