NEW SOUTH WALES

Higher School Certificate

Mathematics Extension 2

Exercise 10/67

by James Coroneos*

1. If $z_1 = 2 + 3i$, $z_2 = -1 + 4i$, show on separate Argand diagrams **(i)** z_1 **(ii)** z_2 **(iii)** $z_1 + z_2$ **(iv)** $z_1 - z_2$ **(v)** $z_2 - z_1$ **(vi)** $z_1 z_2$ **(vii)** iz_1 **(viii)** iz_2

2. Show on separate Argand diagrams the points representing
(i) $2 - i$ **(ii)** $3 + 4i$ **(iii)** $(2 - i) + (3 + 4i)$ **(iv)** $(2 - i) - (3 + 4i)$
(v) $(2 - i)(3 + 4i)$ **(vi)** $i(2 - i)$ **(vii)** $i(3 + 4i)$

3. Verify the triangle inequalities, $|z_1 + z_2| \le |z_1| + |z_2|$, $|z_1 - z_2| \ge |z_1| - |z_2|$ when **(a)** $z_1 = 2 + 3i$, $z_2 = -1 + 4i$ **(b)** $z_1 = 2 - i$, $z_2 = 3 + 4i$.

4. If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$, $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ express $z_1 z_2$ and z_1/z_2 in mod-arg form.
(i) Find the modulus and argument of $z_1 = -2\sqrt{3} + 2i$, $z_2 = 4i$, $Z = \frac{1}{2}(1 + i\sqrt{3})$. Deduce that $Z = z_1/z_2$ and verify this by expressing z_1/z_2 in the form $x + iy$.
(ii) Show that $Z^3 = -1$ and find all the roots of $z^3 = -1$.

5. The points O, A, P_1, P_2, P in an Argand diagram correspond respectively to the complex numbers $0, 1, z_1, z_2, z$. Further, the triangles OP_2P_1, OAP are similar, with vertices corresponding in the order given. Prove that $z = z_1/z_2$.

6. Show by induction that $|z_1 + z_2 + z_3 + \cdots + z_n| \le |z_1| + |z_2| + |z_3| + \cdots + |z_n|$ assuming that $|z_1 + z_2| \le |z_1| + |z_2|$.

*Other resources by James Coroneos are available. Write to P.O. Box 25, Rose Bay, NSW, 2029, Australia, for a catalogue. Typeset by $\mathcal{AMS}$-TEX.