

HSC Trial Examination 2006

General Mathematics

This paper must be kept under strict security and may only be used on or after the morning of Tuesday 8 August, 2006 as specified in the Neap Examination Timetable.

General Instructions

Reading time 5 minutes.

Working time 2½ hours.

Write using black or blue pen.

Calculators may be used.

A formulae sheet is provided at the back of this paper.

Total marks – 100

Section I Pages 2–10 22 marks

- Attempt Questions 1–22.
- Allow about 30 minutes for this section.

Section II Pages 11–20 78 marks

- Attempt Questions 23–28.
- Allow about 2 hours for this section.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2006 HSC General Mathematics Examination.

Section I

Total 22 marks

Attempt Questions 1–22.

Allow about 30 minutes for this section.

Use the multiple-choice answer sheet.

Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A B C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

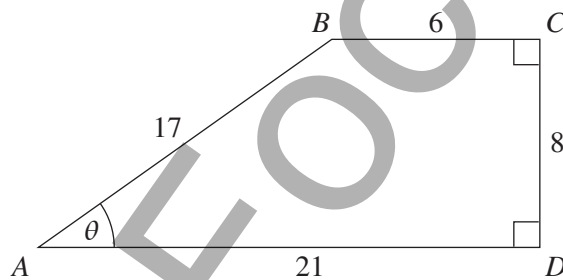
A B C D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and draw an arrow as follows:

correct
↓

A B C D

1. The normal price of a tin of paint is \$20. During a sale prices were reduced by 10%. As a regular customer, Paddy also receives an additional 5% discount off the discounted price. How much will Paddy have to pay for the tin of paint?
- (A) \$17.00
(B) \$17.10
(C) \$18.00
(D) \$18.90
2. Simplify $\frac{20M^8}{4M^2}$.
- (A) $5M^4$
(B) $5M^6$
(C) $16M^4$
(D) $16M^6$
3. Which of the following events is the most likely to happen?
- (A) Tossing a head with a coin.
(B) Rolling a 4 with a normal die.
(C) Correctly guessing the answer to a multiple-choice question with alternatives A , B , C and D .
(D) Choosing a red jelly bean at random from a jar containing 5 red and 10 black jelly beans.
- 4.



NOT
TO
SCALE

In the diagram $ABCD$ is a trapezium. $\angle BCD = 90^\circ$, $\angle CDA = 90^\circ$, $BC = 6$, $CD = 8$, $AD = 21$ and $\angle BAD = \theta$.

What is the value of $\tan \theta$?

- (A) $\frac{8}{15}$
(B) $\frac{8}{17}$
(C) $\frac{10}{17}$
(D) $\frac{10}{21}$

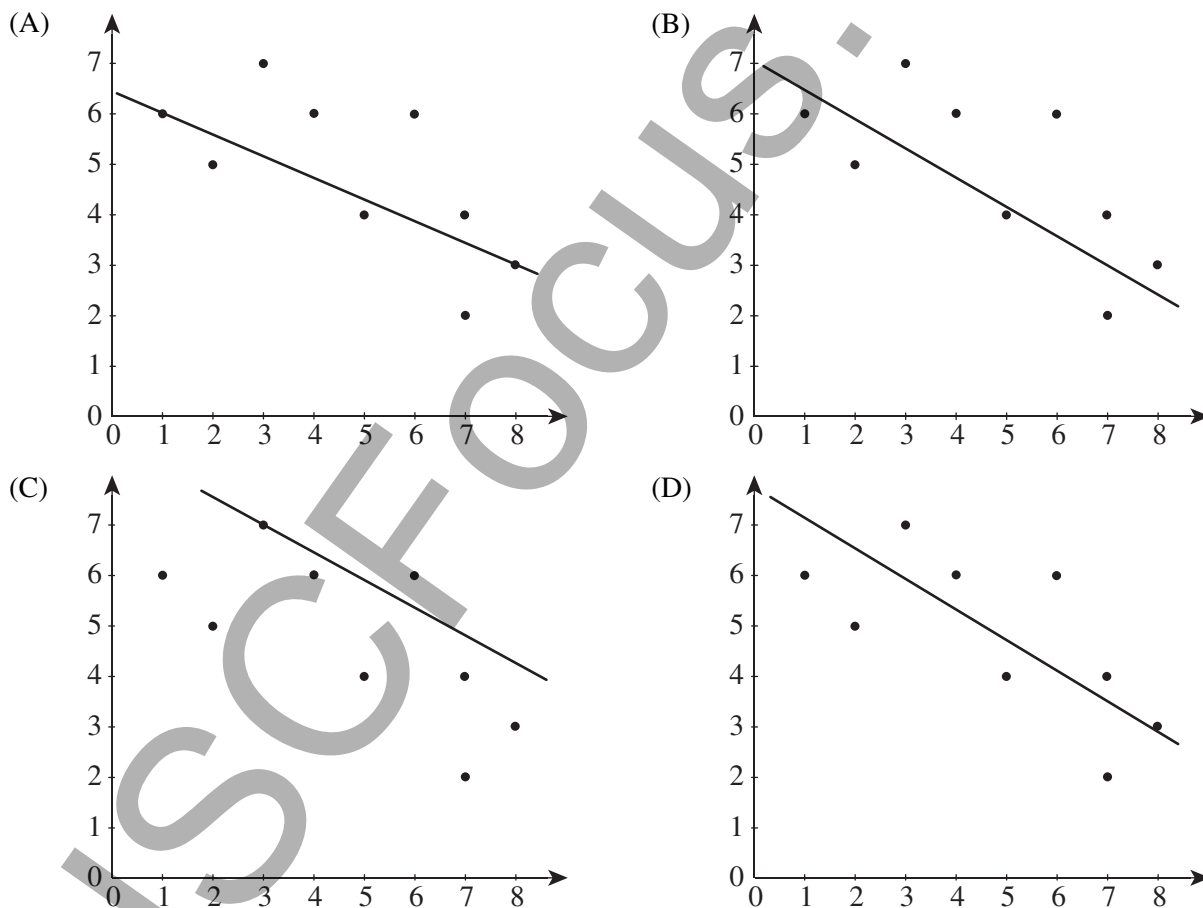
5. If x is the number of years required for an investment to double at 7% p.a. compound interest, then $(1.07)^x = 2$. Eva is trying to solve this equation by ‘guess and check’.

This table shows her progress.

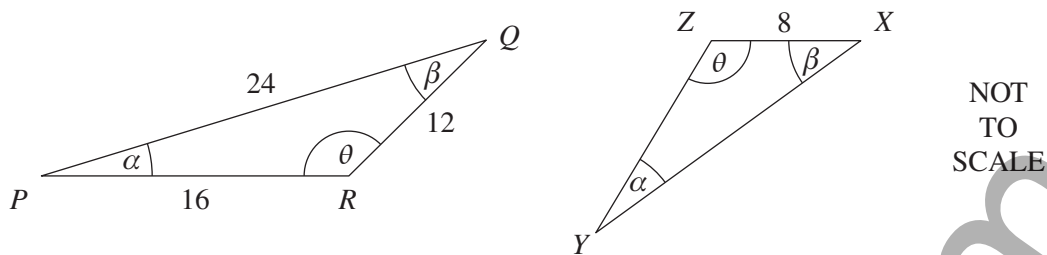
<i>The values Eva tried for x</i>	20	5	12	7	9
<i>The value of $(1.07)^x$ to 2 decimal places</i>	3.87	1.40	2.25	1.61	1.84

Eva’s next try should be between which pair of numbers?

- (A) 5 and 7
 - (B) 7 and 9
 - (C) 9 and 12
 - (D) 12 and 20
6. Which graph best shows the median regression line for the 9 points shown? (The 9 points are the same in each graph.)



7.

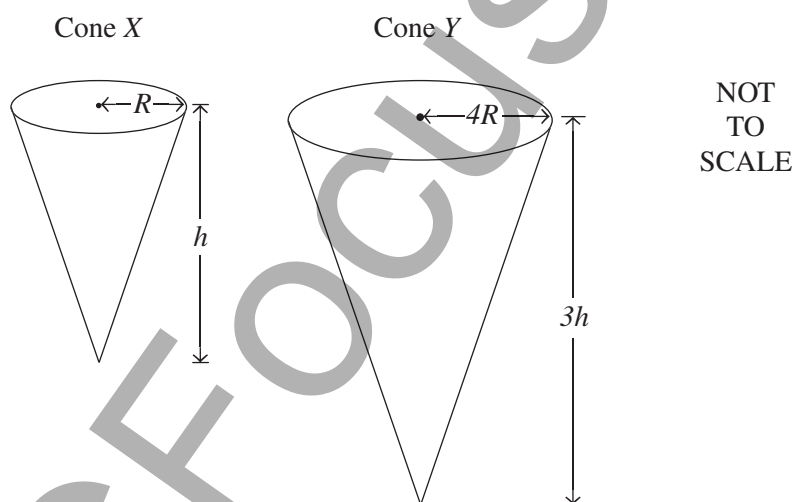


ΔPQR is an enlargement of ΔYXZ .

What is the enlargement, or scale, factor?

- (A) $1\frac{1}{2}$
- (B) 2
- (C) 3
- (D) 8

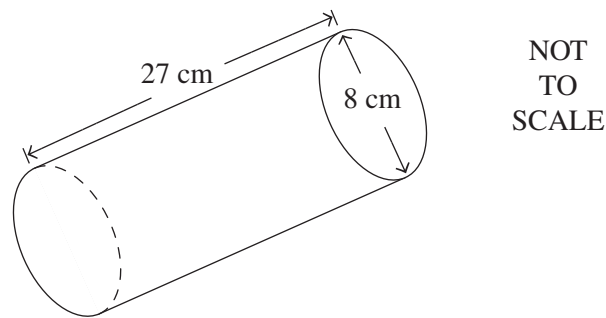
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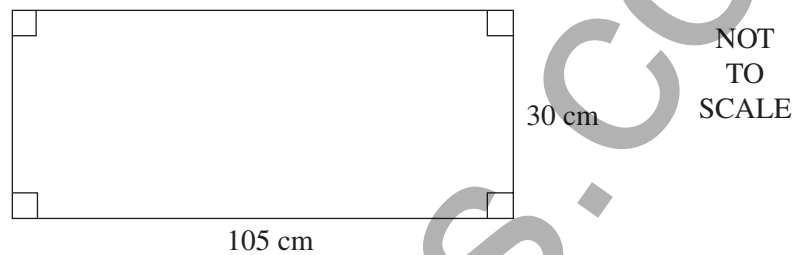
How many times bigger is the volume of cone Y than the volume of cone X?

- (A) 4
- (B) 12
- (C) 48
- (D) 144

9.



Duska is making cylindrical tubes by rolling up cardboard. The tubes have no ends. Duska has one sheet of cardboard 30 cm by 105 cm.



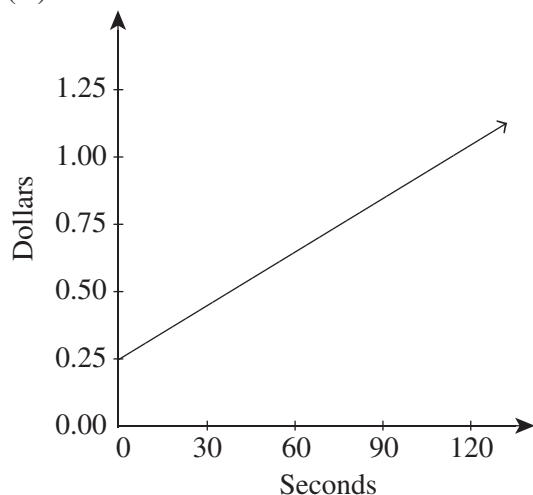
What is the maximum number of tubes Duska can make from the sheet of cardboard?

- (A) 3
 - (B) 4
 - (C) 5
 - (D) 13
10. Meshel was charged an installation fee of $\$F$, and a monthly fee of $\$M$ for her satellite TV service. She calculated the total cost, $\$E$, of the installation and monthly fee for N months using the formula $E = M \times N + F$.
- In this formula, which pronumeral stands for the independent variable?
- (A) E
 - (B) F
 - (C) M
 - (D) N
11. Make S the subject of the formula $V^2 = U^2 - 2aS$.
- (A) $S = \frac{U^2 - V^2}{2a}$
 - (B) $S = \frac{V^2 - U^2}{2a}$
 - (C) $S = U^2 - \frac{V^2}{2a}$
 - (D) $S = V^2 - \frac{U^2}{2a}$

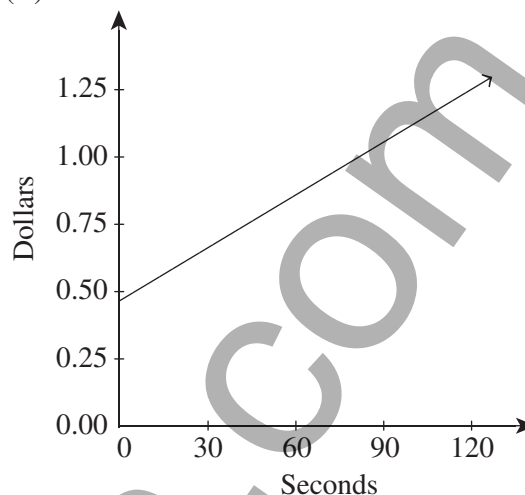
12. Calls costs for Callum's mobile phone consist of a 25 cent connection fee plus 20 cents per 30 seconds or part thereof.

Which graph best displays the cost of Callum's mobile phone calls?

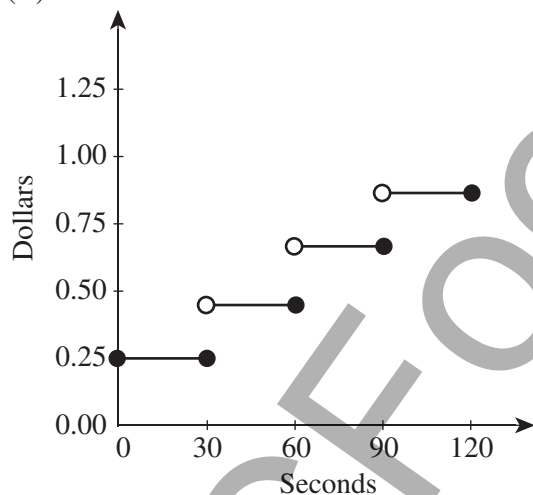
(A)



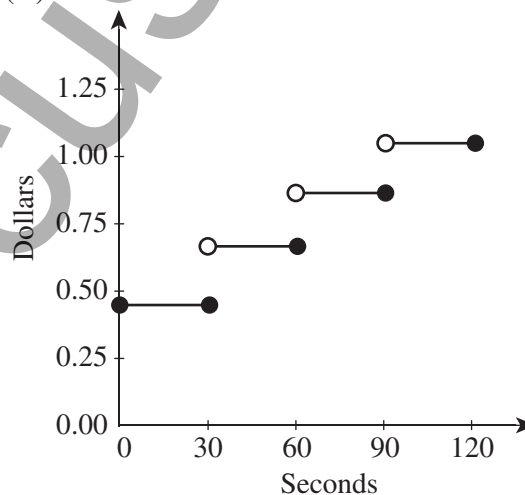
(B)



(C)



(D)



13. There are six students in a year 9 French class. On Monday, when Owen was absent from school, the other five students sat a listening test and obtained scores with a mean value of 12. After Owen completed the test the class's mean score became 11.

What was Owen's mark on the listening test?

- (A) 6
 (B) 8
 (C) 10
 (D) 11

14. The formula $C = \frac{5}{9}(F - 32)$ can be used to convert temperatures between degrees Celsius ($^{\circ}\text{C}$) and degrees Fahrenheit ($^{\circ}\text{F}$).

Which of the following temperatures is equivalent to 50°C ? Answer to the nearest whole degree.

- (A) 10°F
(B) 60°F
(C) 122°F
(D) 148°F
15. Four sheep can eat all the grass in a 1-hectare paddock in 2 days. How many days will it take 5 sheep to eat all the grass in a similar 100-hectare paddock?
- (A) 40
(B) 63
(C) 80
(D) 160
16. The scores of a sample of people are recorded in this table.

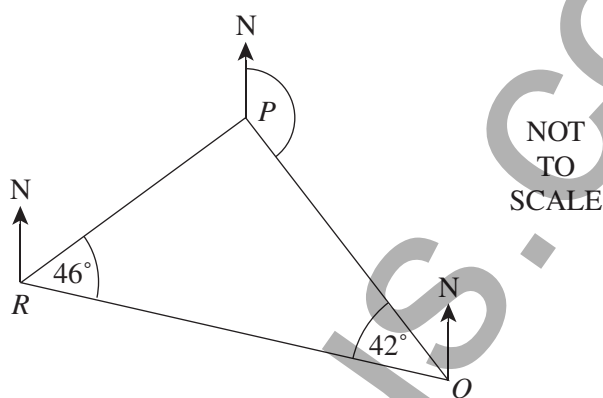
<i>Score</i>	<i>Frequency</i>
6	2
7	3
8	4
9	12
10	9
Total	30

Which of the following statements gives the correct median and distribution for the data listed above?

- (A) The median is 8 and the data is skewed.
(B) The median is 8 and the data is normally distributed.
(C) The median is 9 and the data is skewed.
(D) The median is 9 and the data is normally distributed.
17. Peter is using the capture-recapture technique to estimate the number of fish in a dam. He took 200 fish out of the dam, tagged them, then put the fish back in the dam. Later, Peter caught 50 fish from the dam, 8 of which were tagged. What is the approximate number of fish in the dam?
- (A) 242
(B) 1250
(C) 1600
(D) 80 000

18. Seven properties are for sale in Berkley. The prices are:
\$305 000, \$305 000, \$316 000, \$320 000, \$325 000, \$330 000 and \$1.3 million.
Which statistical measure of these seven prices gives the best indication of the typical property price in Berkley?
- (A) mean
(B) median
(C) mode
(D) range

19.



The diagram shows three towns, P , Q and R , and the direction of north from each town. $\angle PRQ = 46^\circ$ and $\angle RQP = 42^\circ$. The bearing from P to Q is 160° .

What is the bearing of R from P ?

- (A) 072°
(B) 206°
(C) 248°
(D) 252°
20. Narelle needs to give her horse a vitamin supplement at a rate of 20 grams per litre, in its water. The vitamin is available in a solution of 500 grams in 1 litre.
How much of the vitamin solution should Narelle add to the horse's 10 litres of water?
- (A) 100 mL
(B) 250 mL
(C) 400 mL
(D) 5 L
21. Danny has two financial options:
Option 1: A 90% chance of gaining \$400
and a 10% chance of gaining \$200
Option 2: A 70% chance of gaining \$700
and a 30% chance of losing \$200
Which option has the greater financial expectation and by how much?
- (A) Option 1 by \$90.
(B) Option 1 by \$100.
(C) Option 2 by \$50.
(D) Option 2 by \$170.

22.

<i>Score</i>	<i>Frequency</i>	<i>Cumulative frequency</i>
10	7	7
11	16	23
12	6	29
13	4	33

For this set of data, which pair of measures have the same value:
the lower quartile, upper quartile, median and mean?

- (A) The upper quartile and mean.
- (B) The upper quartile and median.
- (C) The lower quartile and mean.
- (D) The lower quartile and median.

END OF SECTION I

Section II

Total marks 78

Attempt Questions 23–28.

Allow about 2 hours for this section.

Answer each question in a SEPARATE writing booklet.

All necessary working should be shown in every question.

-
- | | Marks |
|---|--------|
| Question 23 (13 marks) Use a SEPARATE writing booklet. | |
| (a) Anika is making monthly repayments of \$99.40 for 2 years to repay a \$2000 loan.
How much interest will she pay? | 2 |
| (b) Ian bought a painting at an American art auction.
The painting sold for \$US 800 plus 15% commission.
(i) Calculate the commission in \$US.
(ii) The conversion rate between Australian dollars and US dollars is \$A 1 = \$US 0.75.
Calculate the cost of the painting, including commission, in Australian dollars. | 1
2 |
| (c) How much do you need to invest today at 6% p.a. annually compounding interest to have \$10 000 in 5 years time.
Express your answer correct to the nearest dollar. | 2 |
| (d) Hamish receives a disability pension. The maximum pension is \$488.90 per fortnight, but for every dollar he earns over \$124 per fortnight, Hamish loses 40 cents from his pension. Hamish earns \$290 per fortnight.
Calculate the amount of pension Hamish can receive. | 2 |

Question 23 continues on page 12

Question 23 (Continued)

- (e) Medical research workers have developed a new test for performance enhancing drugs. They are trialling the drug on members of the general community.

The results of the trial are shown in the table.

	<i>Test indicated drugs used</i>	<i>Test indicated drugs not used</i>	<i>Total</i>
<i>People who use drugs</i>	48	7	55
<i>People who don't use drugs</i>	5		
<i>Total</i>	53		120

- (i) Copy the table into your writing booklet and complete the three missing values. **2**
- (ii) For what fraction of the people tested was the test result incorrect? **1**
- (iii) For what percentage of the people who used the drugs did the test indicate that they didn't use drugs? **1**

End of Question 23

Question 24 (13 marks) Use a SEPARATE writing booklet.

- (a) When Mitchell borrowed \$80 000 at 7.2% p.a. monthly compounding interest over 10 years, his monthly repayments were \$937.13.

This table summarises his loan. All entries are correct to the nearest dollar.

<i>Time</i>	<i>Total Mitchell has repaid</i>	<i>Total amount of interest Mitchell has paid</i>	<i>Balance still owing</i>
End of the 1 st year	\$11 246	\$5575	\$74 330
End of the 2 nd year	\$22 491	\$10 729	\$68 237
End of the 3 rd year	<i>X</i>	\$15 428	\$61 692
End of the 4 th year	\$44 982	\$19 641	\$54 659
End of the 5 th year	\$56 228	\$23 330	\$47 102
End of the 6 th year	\$67 473	\$26 457	\$38 984
End of the 7 th year	\$78 719	\$28 980	<i>Y</i>
End of the 8 th year	\$89 964	\$30 854	\$20 889
End of the 9 th year	\$101 240	\$32 030	\$10 819
End of the 10 th year	\$112 455	\$32 456	\$0

- (i) Show that Mitchell paid \$1176 interest during the 9th year. 1
- (ii) Why did he pay over \$5000 more interest in the first year than he did in the 9th year? 1
- (iii) Two values in the table are missing. What values should be in the positions marked *X* and *Y*? 2
- (iv) If interest rates increase and Mitchell leaves his repayments the same, describe how the values in the Balance still owing column will change and give a reason for your answer. 1
- (v) Is it true that even though the interest rate is only 7.2% p.a., almost 30% of Mitchell's repayments will be interest? Use a calculation to support your answer. 1
- (b) Diane invested \$1000 in monthly compounding interest. 2

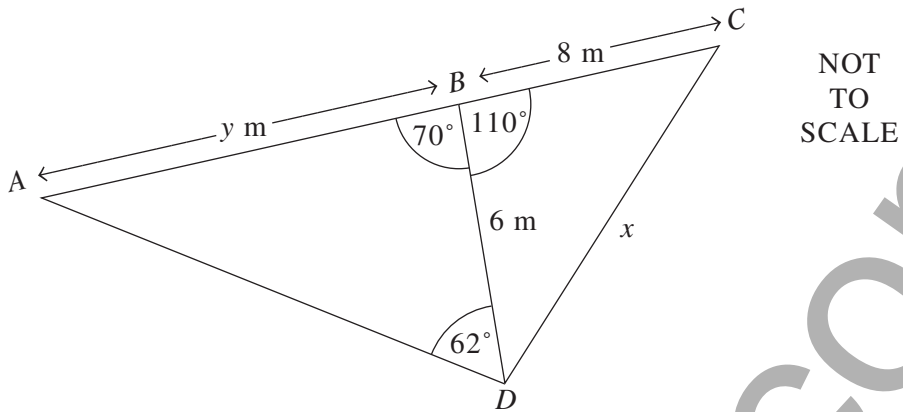
To calculate the amount her investment would grow to by the end of the investment she evaluated 1000×1.007^{48} .

What was the annual rate of interest and the term of the loan?

Question 24 continues on page 14

Question 24 (Continued)

(c)

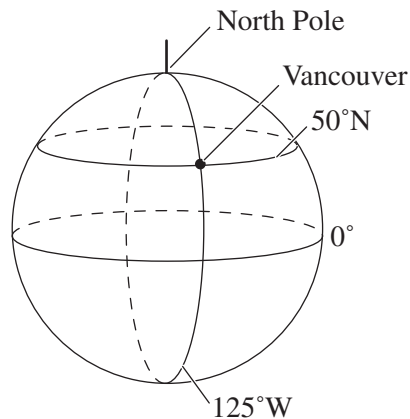


- | | |
|---|----------|
| (i) Determine the value of x and y in this diagram. | 4 |
| (ii) Calculate the area of $\triangle DBC$. | 1 |

End of Question 24

Question 25 (13 marks) Use a SEPARATE writing booklet.

(a)



- (i) The position coordinates of Vancouver are 50°N , 125°W . 2

Calculate the distance from Vancouver to the North Pole in nautical miles.

- (ii) The latitude of the Canadian cities Vancouver and Winnipeg is 50° north and the difference in their longitudes is 30° . Winnipeg is east of Vancouver. 1

What are the position coordinates of Winnipeg?

- (iii) What time and day is it in Vancouver when it is 11 pm Monday in Winnipeg? 2

- (iv) The radius of the 50°N parallel of latitude is 4110 km. 2

Calculate the distance from Vancouver to Winnipeg along the 50°N small circle.

- (b) The probability that a biased coin shows a 'head' is 0.8.

- (i) What is the probability that the biased coin will show a 'tail'? 1

- (ii) David is going to toss the biased coin and a normal coin. 2

What is the probability that one or both of the coins will show 'heads'?

There are 24 coins in a bag. Some of the coins are biased and the remainder are normal. There are k biased coins in the bag.

- (iii) Write an expression involving k for the number of normal coins in the bag. 1

- (iv) When I choose a coin from the bag at random, I am twice as likely to choose a biased coin as I am to choose a normal coin. 2

Write an equation, and solve it, to determine the number of biased coins in the bag.

End of Question 25

Question 26 (13 marks) Use a SEPARATE writing booklet.

- (a) People who fall into cold water often die. The length of time a typical Year 12 girl can survive in cold water can be modelled by the formula

$$S = 16 \times (1.13)^t$$

where S = the survival time in minutes and

t = the temperature of the water in degrees centigrade ($^{\circ}\text{C}$).

- (i) Use the formula to determine the number of minutes a typical Year 12 girl will survive if she falls into water with temperature 0°C . **1**
- (ii) Use the formula $S = 16 \times (1.13)^t$ to determine the missing value in the table. **1**

Temperature, t	4	10	16	21	27
Survival time, S	26		113	208	433

- (iii) Draw a graph that shows the survival time in minutes of a typical Year 12 girl in cold water. Show values from 0°C to 27°C on the horizontal axis. Label the axes and the scales carefully. **4**
- (iv) Which term best describes this model: cubic, exponential, hyperbolic, linear or parabolic? **1**
- (v) Kurrily is a typical Year 12 girl. She was rescued, alive, after she spent 80 minutes in cold water. **1**
- Use the graph you constructed in part iii to determine the minimum temperature the water could have been.
- (vi) Explain any limitations of this model. **1**
- (vii) If a person in cold water tries to swim or tread water, their survival time decreases by 50%. **1**
- On the graph you constructed in part iii, show a second graph to illustrate the survival time of a typical Year 12 girl in cold water if she tries to swim or tread water. Label the graphs carefully.

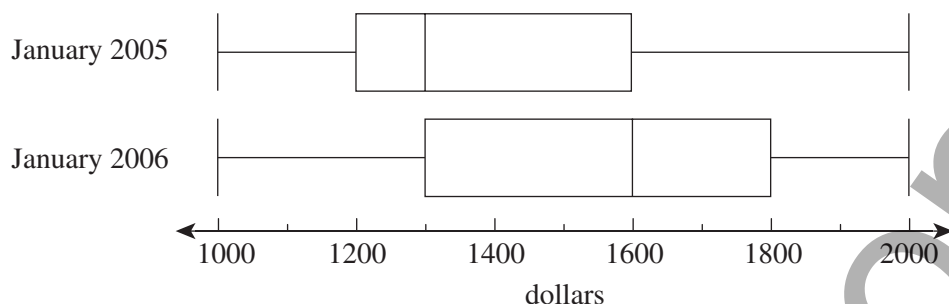
- (b) The number of minutes, T , a person can survive in water with a temperature of 4°C is directly proportional to the square of their mass, M , i.e. $T \propto M^2$. A 60 kg person can survive for 30 minutes in water at 4°C . **3**

How long can a 30 kg child survive in water the same temperature?

End of Question 26

Question 27 (13 marks) Use a SEPARATE writing booklet.

- (a) John owns a muffin business. He displayed the daily total sales for January 2005 and January 2006 in a pair of box-and-whiskers plots.

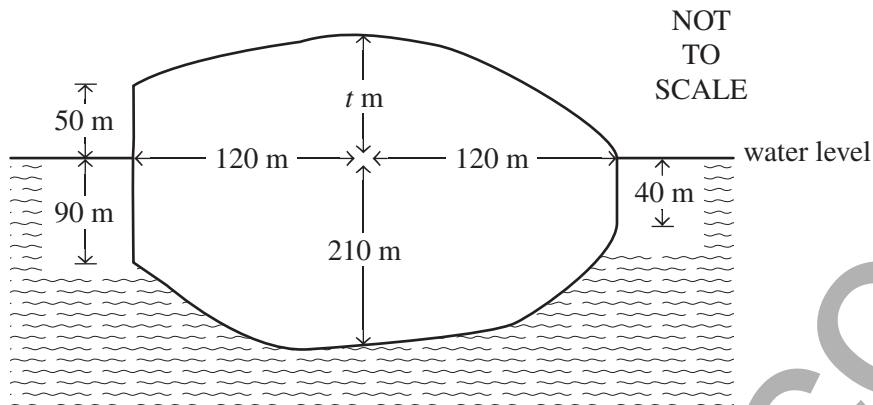


- (i) Compare and contrast the daily sales in January 2005 and January 2006. 2
- (ii) John worked 28 days in January. Estimate his total sales for the month in 2006. Show a calculation to justify your estimate. 1
- (b) The mean weight of golden retriever dogs is 34 kg and the standard deviation is 5 kg.
- (i) The recommended weight for golden retrievers is 24 to 34 kg. 2
- What percentage of golden retrievers are in the recommended weight range?
- (ii) Honey is a golden retriever and her weight has a z-score of 2. How many kilograms does she need to lose to lie within the healthy weight range? 2

Question 27 continues on page 18

Question 27 (Continued)

(c)



The diagram shows the cross-section of an iceberg. The area of the cross-section is $52\,000\text{ m}^2$.

(i) Use two applications of Simpson's Rule to determine the approximate height, t metres, of the top of the iceberg above the water. 3

(ii) The iceberg is in the shape of a regular solid, 500 metres long. 1

Calculate the volume of ice in the iceberg in cubic metres.

(iii) When water freezes to become ice the volume increases by 9%. 2

Determine the number of cubic metres of water that froze to make the iceberg. Express your answer in scientific notation correct to 2 significant figures.

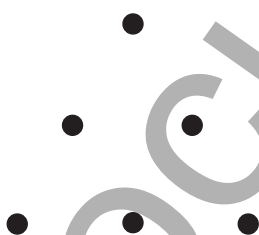
End of Question 27

Question 28 (13 marks) Use a SEPARATE writing booklet.

- (a) Jo is using straight line depreciation to calculate the depreciated value of her business machinery.

<i>New value</i>	\$70 000
<i>Value at 3 years old</i>	\$28 000

- (i) What is the value of the taxation deduction Jo has been able to claim for depreciation each year? **1**
- (ii) Calculate the annual rate of straight line depreciation Jo is using. **1**
- (iii) Calculate the salvage value of the machinery when it is 3 years old if Jo was depreciating it at 25% p.a. with the declining balance method of depreciation. **2**
- (iv) If Jo had chosen to use declining balance depreciation at the rate of 25% p.a. how much could she have claimed off her tax for depreciation in the third year? **2**
- (b) Imran is conducting a psychology experiment. He arranged six lights in an equilateral triangle pattern.



He asks his subjects to select 3 lights and turn them on.

- (i) Show that there are 20 different ways the subjects can choose three lights out of the six lights. **2**
- (ii) How many different equilateral triangles can be made from 3 lights in the pattern? **1**
- (iii) Imran recorded whether the pattern made by the lights the subject turned on made an equilateral triangle. **1**

<i>Lights made an equilateral triangle</i>	30
<i>Lights didn't make an equilateral triangle</i>	50

What is the experimental probability that Imran's subjects will choose to turn on lights that make an equilateral triangle?

Question 28 continues on page 20

Question 28 (Continued)

(iv) Imran's conclusion:

3

“When I ask people to turn on three lights, they don't choose the lights at random.”

Do you agree with Imran's conclusion? Support your answer with a calculation of the theoretical probability that three randomly selected lights in the pattern will form an equilateral triangle.

END OF PAPER

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Formulae Sheet

Area of an annulus

$$A = \pi(R^2 - r^2)$$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$$A = \pi ab$$

a = length of semi-major axis

b = length of semi-minor axis

Area of a sector

$$A = \frac{\theta}{360} \pi r^2$$

θ = number of degrees in central angle

Arc length of a circle

$$l = \frac{\theta}{360} 2\pi r$$

θ = number of degrees in central angle

Simpson's rule for area approximation

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Surface area

Sphere:

$$A = 4\pi r^2$$

Closed cylinder:

$$A = 2\pi rh + 2\pi r^2$$

r = radius

h = perpendicular height

Volume

Cone:

$$V = \frac{1}{3}\pi r^2 h$$

Cylinder:

$$V = \pi r^2 h$$

Pyramid:

$$V = \frac{1}{3}Ah$$

Sphere:

$$V = \frac{4}{3}\pi r^3$$

r = radius

h = perpendicular height

A = area of base

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle

$$A = \frac{1}{2}ab \sin C$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

OR

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Simple interest

$$I = Prn$$

- P = initial quantity
- r = percentage interest rate per period, expressed as a decimal
- n = number of periods

Compound interest

$$A = P(1 + r)^n$$

- A = final balance
- P = initial quantity
- n = number of compounding periods
- r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M \left\{ \frac{(1 + r)^n - 1}{r} \right\}$$

- M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M \left\{ \frac{(1 + r)^n - 1}{r(1 + r)^n} \right\}$$

OR

$$N = \frac{A}{(1 + r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

- S = salvage value of asset after n periods
- V_0 = purchase price of the asset
- D = amount of depreciation apportioned per period
- n = number of periods

Declining balance formula for depreciation

$$S = V_0(1 - r)^n$$

- S = salvage value of asset after n periods
- r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

OR

$$\bar{x} = \frac{\sum fx}{\sum f}$$

- \bar{x} = mean
- x = individual score
- n = number of scores
- f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

- s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of straight line

$$y = mx + b$$

- m = gradient
- b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$