

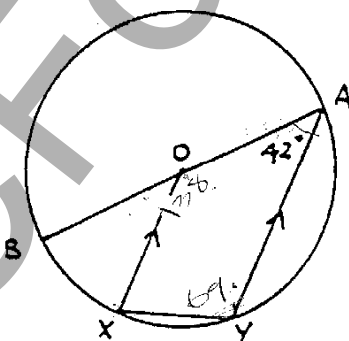
**QUESTION ONE** (Start a new page)

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|--|--------------|
|  | <b>Marks</b> |
| A. In group A there are 5 men and 3 women. In group B there are 6 women and 4 men.   |              |
| (i) If one person is chosen at random from each group what is the probability that the people chosen are of opposite sexes ?         | 2            |
| (ii) If a group, and then one person from that group is chosen at random, what is the probability that the person chosen was a man ? | 2            |
| B) Show that $\tan x = \frac{\sin x}{\cos x}$ Hence, find k to 3 decimal places, given that $\int_0^k \tan x \, dx = 1$              | 5            |
| C) Using the substitution $u = \log_e x$ , evaluate $\int_1^e \frac{\log_e x}{x} \, dx$  | 3            |

**QUESTION TWO** (start a new page)

- (A) The area between the curve  $y = \sin x$ , the x axis and the ordinates  $x = 0$  and  $x = \frac{3\pi}{4}$  is revolved about the x axis. Find the volume of the solid so formed, leaving your answer in exact form.

(B)



O is the centre of the circle. AY is parallel to OX. Angle OAY measures  $42^\circ$ . Find the measure of angle XYA, giving reasons for your answer.

- (C) A student can borrow 4 books from the library. he wants to read 3 history books, 2 romances, 4 science fiction and 3 murder mysteries.
- (i) How many different selections of 4 books can be made ?
- (ii) How many selections can be made if he takes both romances ?
- (iii) How many selections can be made if he decides to take 2 history books ?

3

4

**QUESTION THREE** (Start a new page)**Marks**(A) Consider the function  $f(x) = 3\sin^{-1}\left(\frac{x}{2}\right)$ (i) Find the value of  $f(2)$ (ii) Draw the graph of  $y = f(x)$ 

(iii) State the domain and range of this function

(iv) Find the slope of the curve  $y = f(x)$  at  $x = 0$ 

6

(B)  $\int_0^{1/3} \frac{4}{\sqrt{4-9x^2}} dx$ 

3

(C) Express  $\sqrt{3} \sin x - \cos x$  in the form  $n \sin(x - B)$  and hence solve the equation  $n \sin(x - B) = 1$  in the region  $0 \leq x \leq \pi$ 

3

**QUESTION FOUR** (Start a new page)(A) If  $a, b$  and  $c$  are the roots of  $x^3 - 3x + 2 = 0$ , find  $a^2 + b^2 + c^2$ 

2

(B) The position at time  $t$  of a particle moving along the  $x$ -axis is given by  $x = 2t^3 - 9t^2 + 12t$ . When and where does the particle first come to rest?

3

(C) What is the coefficient of  $x^{-3}$  in the expansion of  $(2x - \frac{1}{2x^2})^9$ 

4

(D) Prove the differentiation of  $y = \sin^{-1} x$ 

2

(E) Find  $\lim_{x \rightarrow 0} \frac{\sin 3x}{\tan \frac{1}{2}x}$ 

2

**QUESTION FIVE** (Start a new page)

- (A) Prove by mathematical induction that :

$$\frac{1}{1(4)} + \frac{1}{4(7)} + \frac{1}{7(10)} + \frac{1}{1(3n-2)(3n+1)} = \frac{n}{3n+1}$$

- (B) (i) Write down the value of
- ${}^n C_k - {}^n C_{n-k}$

- (ii) By comparing the coefficients of
- $x^6$
- on both sides of the identity
- $(1+x)^6 (1+x)^6 = (1+x)^{12}$
- or otherwise, show that :

$$\sum_{k=0}^6 ({}^6 C_k)^2 = {}^{12} C_6$$

- (C) Find the equation of the normal to the curve
- $y = \operatorname{cosec} 3x$
- at the point P with x coordinate
- $\frac{\pi}{4}$

**QUESTION SIX** (Start a new page)

- (A) For the curve
- $y = \frac{x+2}{(x+1)(x-3)}$

- (i) Find the x and y intercepts 2
- (ii) Find any asymptotes 2
- (iii) Find any stationary points and determine their nature 3
- (iv) find any points of inflexion 3
- (v) Sketch the curve 2

**QUESTION SEVEN** (Start a new page)**Marks**

- (A) (i) Derive the equation of the tangent to the parabola whose parametric equation is  $x = 2at$   $y = at^2$ .
- (ii) P and Q are two points on the parabola above with parameters p and q respectively: If the tangents at P and Q meet at T find the co-ordinates of T in terms of p and q.
- (iii) If T lies on the line  $x + y + 5a = 0$  find the relationship between p and q. Hence or otherwise find the locus of the Midpoint M of PQ as P moves around the parabola.

6

- (B) A spherical balloon is being inflated and its radius is increasing at the constant rate of 3 cm/min. At what rate is its volume increasing when the radius of the balloon is 5 cm ?

2

- (C) For the time interval  $0 \leq t \leq \frac{1}{2}$  the velocity of a particle is given by :

$$v = \frac{10}{\sqrt{1-t^2}} + \frac{1}{(1-t)^2} \quad (\text{the units are metres, seconds})$$

4

During the given time interval find :

- (i) The distance travelled by the particle.
- (ii) The maximum velocity attained.