YEAR 11 EXT 1

ASSESSMENT 3.

1. $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

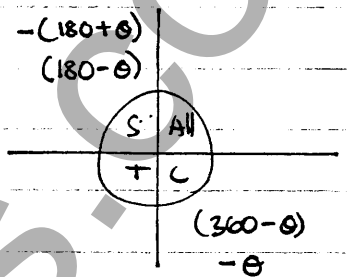
2. $\tan 2\theta = -\sqrt{3}$

$-360^\circ \leq \theta \leq 360^\circ$

$2\theta = -(180+60)^\circ, -60^\circ, (180-60)^\circ, (360-60)^\circ$

$2\theta = -240^\circ, -60^\circ, 120^\circ, 300^\circ$

$\theta = -120^\circ, -30^\circ, 60^\circ, 150^\circ$



3. a) $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

$$\text{LHS} = \frac{2\sin x \cos x}{1 + \cos^2 x - \sin^2 x}$$

$$= \frac{2\sin x \cos x}{1 + 2\cos^2 x - 1}$$

$$= \frac{2\sin x \cos x}{2\cos^2 x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$= \text{RHS}$$

b) $\therefore \tan 15^\circ = \dfrac{\sin 30^\circ}{1 + \cos 30^\circ}$

$$= \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{1}{2}}{\left(\frac{2+\sqrt{3}}{2}\right)}$$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{4-3}$$

$$= 2-\sqrt{3}$$

4. LHS = $\dfrac{1-\cos\theta}{\sin\theta}$

$$= \frac{1-\left(\frac{1-t^2}{1+t^2}\right)}{\frac{2t}{1+t^2}}$$

$$= \frac{\left(\frac{1+t^2-1+t^2}{1+t^2}\right)}{\frac{2t}{1+t^2}}$$

$$= \frac{2t^2}{2t}$$

$$= t$$

RHS = $\dfrac{\sin\theta}{1+\cos\theta}$

$$= \frac{\left(\frac{2t}{1+t^2}\right)}{\left(1+\frac{1-t^2}{1+t^2}\right)}$$

$$= \frac{\left(\frac{2t}{1+t^2}\right)}{\left(\frac{1+t^2+1-t^2}{1+t^2}\right)}$$

$$= \frac{2t}{2}$$

$$= t$$

LHS = RHS

Section 2

1 $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(\cos x - 1)(2\cos x + 1) = 0$

$\cos x = 1$ or $2\cos x + 1 = 0$

$x = 0°, 360°$ $2\cos x = -1$

$\cos x = \frac{-1}{2}$

1. continued.

$\cos x = -\frac{1}{2}$

$x = (180-60)^\circ, (180+60)^\circ$

$x = 120^\circ, 240^\circ$

∴ solution is $x = 0^\circ, 120^\circ, 240^\circ, 360^\circ$

2. a) $\cos x - \sqrt{3}\sin x = R\cos(x+\alpha)$

$R = \sqrt{1^2 + (-\sqrt{3})^2}$ $\qquad \tan\alpha = \frac{\sqrt{3}}{1}$

$= 2.$ $\qquad \alpha = 60^\circ$

∴ $\cos x - \sqrt{3}\sin x = 2\cos(x+60^\circ)$

b) $\cos x - \sqrt{3}\sin x = 1$ $\qquad 60^\circ \le x \le 420^\circ$

$2\cos(x+60^\circ) = 1$

$\cos(x+60^\circ) = \frac{1}{2}$

$x+60^\circ = 60^\circ, (360^\circ - 60^\circ), (360+60)^\circ$

$x+60^\circ = 60^\circ, 300^\circ, 420^\circ$

$x = 0^\circ, 240^\circ, 360^\circ$

3. a) In Δ BDC:

$\tan 27^\circ = \frac{1545}{BC}$

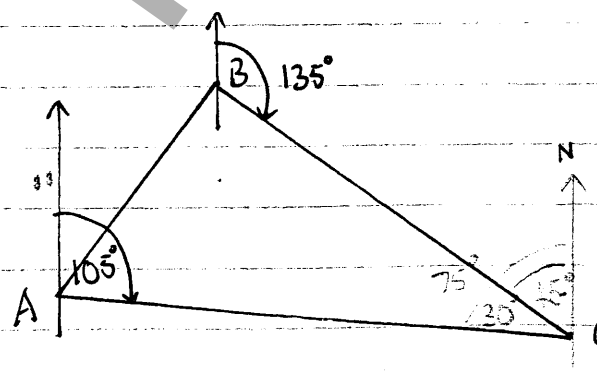
$BC = \frac{1545}{\tan 27^\circ}$

In Δ ADC:

$\tan 22^\circ = \frac{1545}{AC}$

$AC = \frac{1545}{\tan 22^\circ}$

In Δ ABC:



$\angle BCN = 45^\circ$

$\angle ACN = 75^\circ$

$\angle ACB = 75^\circ - 45^\circ = 30^\circ$

3 continued

$$AB^2 = BC^2 + AC^2 - 2 \times BC \times AC \times \cos 30^\circ$$

$$= \left(\frac{1545}{\tan 27^\circ}\right)^2 + \left(\frac{1545}{\tan 22^\circ}\right)^2 - 2 \times \left(\frac{1545}{\tan 27^\circ}\right) \times \left(\frac{1545}{\tan 22^\circ}\right) \times \cos 30^\circ$$

$$= 3\,733\,857.152$$

$$AB = 1932.319112$$

$$AB = 1932 \text{ m (to nearest m)}$$

b). Let $\angle ABC = \theta$.

$$\frac{\sin\theta}{AC} = \frac{\sin 30^\circ}{AB}$$

$$\sin\theta = \frac{\sin 30^\circ}{AB} \times AC$$

$$= \frac{\sin 30^\circ}{1932} \times \frac{1545}{\tan 22^\circ}$$

$$= 0.984\,486\,475\ldots$$

$$\theta = 81^\circ\ 41'$$

$\therefore$ bearing is $135^\circ + 82^\circ = 217^\circ$

Section 2.

1.

A(-3, 5) ——— B(2, -1)

2 : -3

$x = \frac{2(2) + (-3)(-3)}{2-3}$ $\quad y = \frac{2(-1) + (-3)(5)}{2-3}$

$= \frac{4+9}{-1}$ $\quad = \frac{-2-15}{-1}$

$= -13$ $\quad = 17$

$\therefore P(-13, 17)$

2. $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$\frac{2\sqrt{10}}{5} = \frac{|k + 3(0) + 2|}{\sqrt{1^2+3^2}}$

$\frac{2\sqrt{10}}{5} = \frac{|k+2|}{\sqrt{10}}$

$\frac{2 \times 10}{5} = |k+2|$

$|k+2| = 4$

$k = 4-2$ or $k = -4-2$

$k = 2$ $\quad k = -6$

3. $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$2x - y + 1 = 0$ $\quad x - 3y + 4 = 0$

$y = 2x + 1$ $\quad x + 4 = 3y$

$m_1 = 2$ $\quad y = \frac{1}{3}x + \frac{4}{3}$

$m_2 = \frac{1}{3}$

$$\tan\alpha = \left|\frac{2-\frac{1}{3}}{1+2\times\frac{1}{3}}\right|$$

$$= 1$$

$$\alpha = 45^\circ$$

4. Midpoint:

$$x_1 = \frac{0+6}{2} \qquad y = \frac{-3+-1}{2}$$

$$= 3 \qquad y = -2$$

$$\therefore (3,-2)$$

$$m_1 = \tan 60^\circ = \sqrt{3}$$

$$m_2 = \frac{-1}{\sqrt{3}}$$

$$y - y_1 = m(x - x_1)$$

$$y + 2 = \frac{-1}{\sqrt{3}}(x-3)$$

$$\sqrt{3}(y+2) = -(x-3)$$

$$\sqrt{3}y + 2\sqrt{3} = -x + 3$$

$$x + \sqrt{3}y + 2\sqrt{3} - 3 = 0$$