* Follow through with errors.

Year 11 Ext 1

Assess 1. 2005.

1. a) (i) $27x^3 - 8$

$= (3x)^3 - 2^3$

$= (3x-2)(9x^2+6x+4)$ /1

(ii) $2x^3 - x^2y - 21xy^2$

$= x(2x^2 - xy - 21y^2)$ 1

$= x(2x-7y)(x+3y)$ 1 /2

b) (i) $a^2 - 8a + 16$

$= (a-4)(a-4)$ 1

$= (a-4)^2$ /1

(ii) $a^2 - 8a + 16 - b^2$

$= (a-4)^2 - b^2$

$= (a-4-b)(a-4+b)$ 1 /1

c) $\dfrac{5\sqrt{3}+2}{5\sqrt{3}-2} \times \dfrac{5\sqrt{3}+2}{5\sqrt{3}+2}$ 1

$= \dfrac{75 + 20\sqrt{3} + 4}{75 - 4}$

$= \dfrac{79 + 20\sqrt{3}}{71}$ 1 /2

2. a) (i) $|2p+1| = 5$

$2p+1 = 5$ or $2p+1 = -5$

$2p = 4$ $\qquad$ $2p = -6$

$p = 2$ $\qquad$ $p = -3$

1 $\qquad$ 1 /2

2a (ii) $\dfrac{2x+3}{x-1} \geq 1, \quad x \neq 1$

$$(x-1)^{\cancel{2}}\frac{(2x+3)}{(\cancel{x-1})} \geq 1(x-1)^2$$ 1 $\times (x-1)^2$

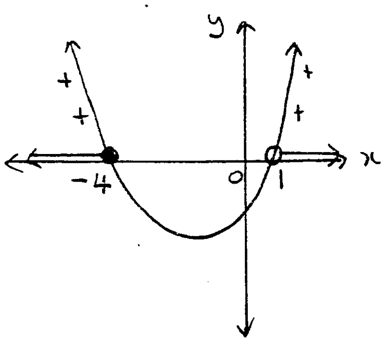
$$2x^2 + x - 3 \geq x^2 - 2x + 1$$

$$x^2 + 3x - 4 \geq 0$$

If $x^2 + 3x - 4 = 0$

$$(x+4)(x-1) = 0$$

$$x = -4 \text{ or } x = 1$$ 1



$\therefore$ solution is $x \leq -4, \; x > 1$ 1 /3

(iii). $x^{-\frac{2}{3}} = \frac{1}{25}$

$$\left(x^{-\frac{2}{3}}\right)^{-\frac{3}{2}} = \left(\frac{1}{25}\right)^{-\frac{3}{2}}$$

$$x = 25^{\frac{3}{2}}$$ 1 or $x = \left(\frac{1}{25}\right)^{-\frac{3}{2}}$

$$x = (\sqrt{25})^3$$

$$x = 125$$ 1 /2

b) $x + y = 2$ —— ①

$5x + y = 6$ —— ②

① - ② $4x = 4$

$x = 1$ 1

Sub into ①

$1 + y = 2$

$y = 1$ 1 /2

$\therefore x = 1, y = 1$

2 c). $xy = 2 \quad — ①$

$3x - y + 1 = 0 \quad — ②$

In ① $\quad y = \frac{2}{x} \quad — ③$

Sub into ②

$3x - \frac{2}{x} + 1 = 0$ (1)

$3x^2 - 2 + x = 0$

$3x^2 + x - 2 = 0$

$(x+1)(3x-2) = 0$

$x + 1 = 0$ or $3x - 2 = 0$

$x = -1 \qquad 3x = 2$

$x = \frac{2}{3}$ (1)

Sub into ③ $\quad x = -1, \; y = \frac{2}{-1} = -2$

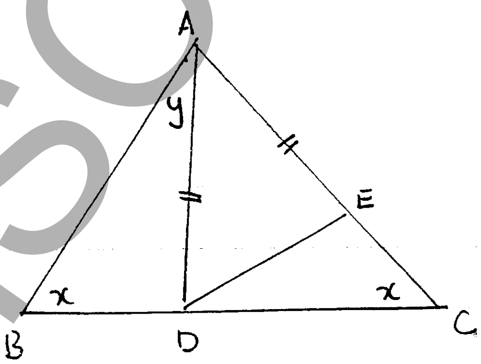
$x = \frac{2}{3}, \; y = \frac{2}{\left(\frac{2}{3}\right)} = 3$ (1)

/3

$\therefore$ solutions are $x = -1, \; y = -2$

$x = \frac{2}{3}, \; y = 3$

3. a).



(i). ext. $\angle$ of $\triangle ABD$ is equal to sum of two opposite interior $\angle$s.

/1

(ii). $\angle DAC = 180 - x - (x+y)$

$= 180 - 2x - y$ } either (1)

/1

3 (a) (iii) $\angle ADE = \dfrac{180 - \angle DAC}{2}$

$$= \frac{180 - (180 - 2x - y)}{2}$$

$$= \frac{180 - 180 + 2x + y}{2}$$

$$= \frac{2x + y}{2}$$

1 any answer

$\angle EDC = \angle ADC - \angle ADE$

$$= x + y - \left(\frac{2x+y}{2}\right)$$

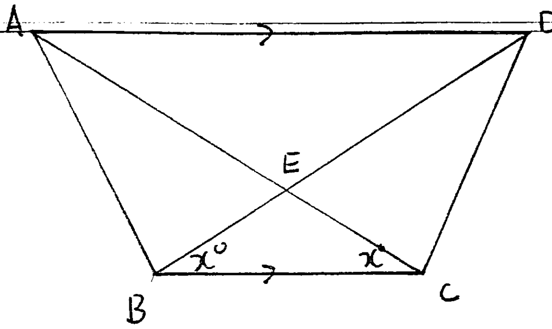
$$= \frac{2(x+y) - (2x+y)}{2}$$

$$= \frac{2x + 2y - 2x - y}{2}$$

$$= \frac{y}{2}$$ 1

/2

3\. b)



(i). $\angle DAE = x$ (alt $\angle$s equal, $AD \parallel BC$)

$\angle ADE = x$ (alt $\angle$s equal, $AD \parallel BC$) } 1

$\therefore$ $\triangle ADE$ is isosceles (base $\angle$s equal)

$\therefore$ $AE = DE$ (equal sides of isos. $\triangle ADE$) 1 /2

(ii) S. $AE = DE$ (proven in (i))

$EC = EB$ (equal sides of isos $\triangle EBC$)

$\therefore$ $AE + EC = DE + EB$

$AC = DB$ } 1

A. $\angle ACB = \angle DBC$ (given) 1

S. BC is common 1

$\therefore$ $\triangle ABC \equiv \triangle DCB$ by SAS /3

4\. a) $f(1) = \sqrt{1-1^2} = 0$

$f(-2) = (-2)^2 + 6(-2) + 5 = -3$ } 1 both correct

$\therefore$ $f(1) - f(-2)$

$= 0 - -3$

$= 3$ 1 /2

4 b). $f(t) = (t-3)^2 = 4$ 1

$t - 3 = \pm 2$

$t - 3 = 2$ or $t - 3 = -2$

$t = 5$ $t = 1$ 1

/2

c) $f(x) = \dfrac{x^2 - 3}{x}$

Test for an odd function is $f(-x) = -f(x)$ 1

LHS $= f(-x) = \dfrac{(-x)^2 - 3}{-x} = \dfrac{x^2 - 3}{-x}$

RHS $= -f(x) = -\left[\dfrac{x^2 - 3}{x}\right] = \dfrac{x^2 - 3}{-x}$

LHS = RHS 1

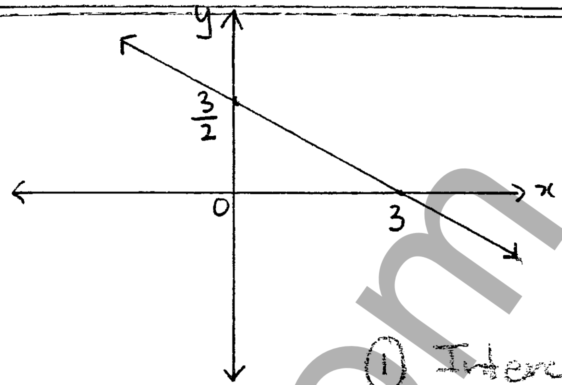
$\therefore$ the function is odd.

/2

4 d). (i) $x + 2y - 3 = 0$

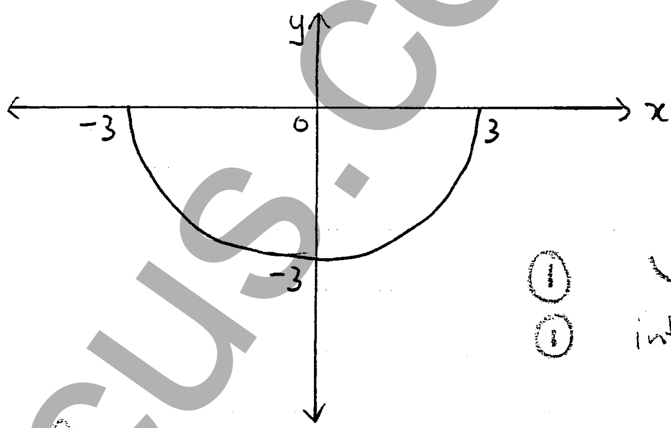
$x = 0, \quad y = \frac{3}{2}$

$y = 0, \quad x = 3$



1 Intercepts

(ii) $y = -\sqrt{9 - x^2}$

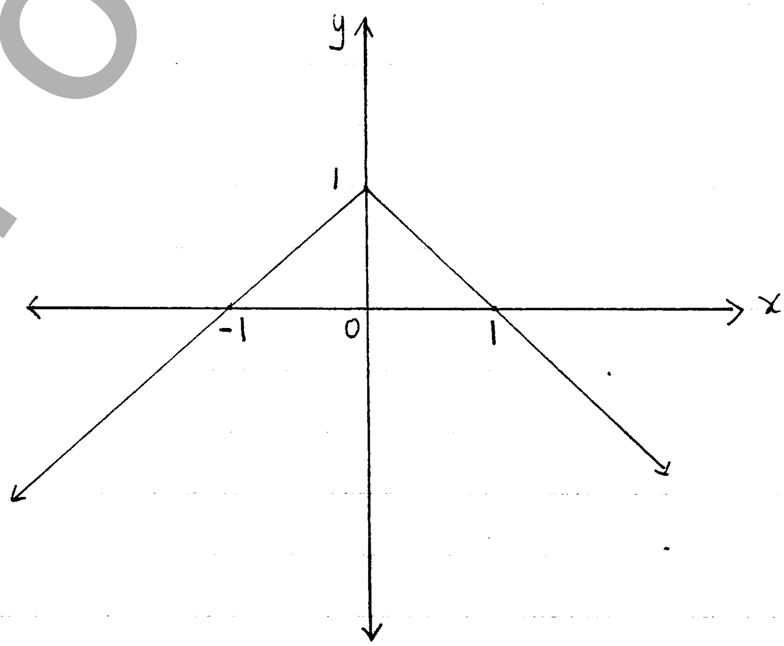


1

1 intercepts

Range: $-3 \le y \le 0$ 1

(iii) $y = 1 - |x|$



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1 intercepts