Year 11 Extension 1
Pre-lim 2005

Question 1

(a) $m_1 = 5 \quad m_2 = 2$

$$\therefore \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{5-2}{1+5\times 2}\right|$$

$$= \frac{3}{11}$$

$$\theta \doteqdot 15^\circ 15' \text{ (nearest minute)}$$

1 – both gradients correct
1 – substitution of 'their' gradients
1 – calculation.

(b) $x_1 = -5 \quad x_2 = 1 \quad m = 3$

$y_1 = 6 \quad y_2 = 0 \quad n = -1$

$$x = \frac{mx_2 + nx_1}{m+n} \qquad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{3(1) - 1(-5)}{3-1} \qquad = \frac{3(0) - 1(6)}{3-1}$$

$$= 4 \qquad = -3$$

$$\therefore P(4, -3)$$

1 – making n neg.
1 – substitutions
1 – answer

(c) $5x - y = 3 \quad (1)$

$2x - y = 0 \quad (2)$

(1) − (2)

$3x = 3 \qquad \therefore 5 - y = 3$

$x = 1 \qquad y = 2$

pt intersection $(1, 2)$

$m = -2$

$\therefore y - 2 = -2(x - 1)$

$y - 2 = -2x + 2$

$y = -2x + 4$

or

$2x + y - 4 = 0$

1 – pt of intersection
1 – correct gradient
1 – 'their' equation.

(d) In $3x - 2y + 1 = 0$

Let $x = 0$ $2y = 1$

$y = \frac{1}{2}$

$\therefore$ A point is $(0, \frac{1}{2})$

$a = 3$ $b = -2$ $c = 3$

$\therefore d = \left| \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

$= \left| \dfrac{3(0) - 2(\frac{1}{2}) + 3}{\sqrt{9 + 4}} \right|$

$= \left| \dfrac{2}{\sqrt{13}} \right|$

$= \dfrac{2}{\sqrt{13}}$ or $\dfrac{2\sqrt{13}}{13}$ units.

1 – finding a correct pt on a line

1 – their substitution

1 – answer

Question 2

(a) $\sin 75 = \sin(45 + 30)$

$= \sin 45 \cos 30 + \cos 45 \sin 30$

$= \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \cdot \dfrac{1}{2}$

$= \dfrac{\sqrt{3} + 1}{2\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{6} + \sqrt{2}}{4}$

1 – expansion of $\sin(45 + 30)$

1 – answer (show the answer!)

(b) $\cos x = \dfrac{1 - t^2}{1 + t^2}$ $\tan x = \dfrac{2t}{1 - t^2}$

$\therefore \sec x + \tan x$

$= \dfrac{1 + t^2}{1 - t^2} + \dfrac{2t}{1 - t^2}$

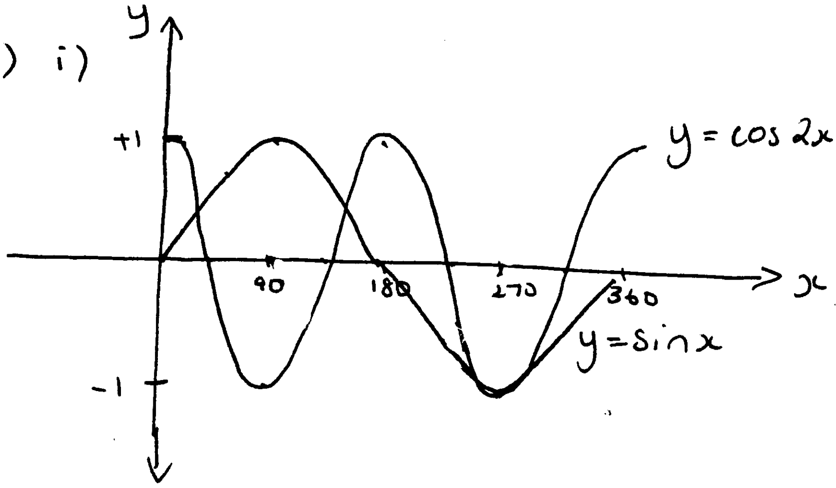
$= \dfrac{t^2 + 2t + 1}{1 - t^2}$

$= \dfrac{(t + 1)^2}{(1 - t)(1 + t)} = \dfrac{t + 1}{1 - t}$

1 – correct substitution including $\sec x$!

1 – simplify fully.

(c) i)



1 - correct sine curve

1 - correct no. of solns from 'their' graph.

3 solutions.

ii) $\cos 2x = \sin x$

$1 - 2\sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\therefore \sin x = \frac{1}{2} \quad \sin x = -1$

$\therefore x = 30°, 150°, 270°$

1 - correct sub. of cos 2x to get quad.

1 - factorising

1 - answers

(d) $LHS = \dfrac{\sin 2\theta}{\sqrt{4 - 4\sin^2\theta}}$

$= \dfrac{2\sin\theta\cos\theta}{\sqrt{4(1-\sin^2\theta)}}$

$= \dfrac{2\sin\theta\cos\theta}{2\sqrt{\cos^2\theta}}$

$= \dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cancel{\cos\theta}}$

$= \sin\theta$

$= RHS.$

1 - correct sub of sin 2θ

1 - factorisation of denominator

1 - fully simplify

Question 3

(a) $2x^2 - 5x + 7 \equiv p(x^2 - 2x + 1) + qx - q + r$

$= px^2 - 2px + p + qx - q + r$ 1 - expansion

$= px^2 - x(2p - q) + p - q + r$ 1 - factorisation

$\therefore p = 2 \quad 2p - q = 5 \quad p - q + r = 7$ 1 - all answers correct.

$4 - q = 5 \quad 2 - -1 + r = 7$

$q = -1 \quad r = 4$

(b) i) $\alpha + \beta = -b/a$

$= --15/3$

$= 5$ 1 - correct only

ii) $\alpha\beta = c/a$

$= 7/3$ 1 - correct only

iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 25 - 14/3$ 1 - correct substitution

$= 20\frac{1}{3}$ or $61/3$.

(c) Let roots be $\alpha, \alpha + 1$

$\therefore 2\alpha + 1 = -b/a \qquad \alpha(\alpha + 1) = c/a$ 1 - sum of roots eqn

$= 1 - 2k \qquad = k + 3$

$\therefore 2\alpha = -2k \qquad \therefore \alpha^2 + \alpha = k + 3 \quad (2)$ 1 - product of roots eqn

$\alpha = -k \quad (1)$ 1 - answers.

Sub $\alpha = -k$ into (2) :

$(-k)^2 + (-k) = k + 3$

$k^2 - k = k + 3$

$k^2 - 2k - 3 = 0$

$(k - 3)(k + 1) = 0$

$k = 3, -1$

(d) $(x-2)^2 \cdot \frac{2x+3}{x-2} \le (x-2)^2$, $\quad x \neq 2$.

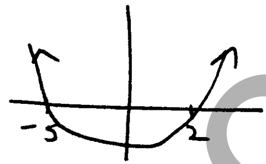
$(x-2)(2x+3) \le (x-2)^2$

$(x-2)(2x+3) - (x-2)^2 \le 0$

$(x-2)\left[2x+3-(x-2)\right] \le 0$

$(x-2)(x+5) \le 0$

1 – quadratic
1 – factorise
1 – answer, must have <2 only!



$-5 \le x < 2$

Question 4

(a) i) D: all real x — 1 – correct only

ii) R: $f(x) \ge 2$ — 1 – correct only

(b) i) $f(x) = \frac{x}{x^2-9}$ — 1 – correct only

$f(-x) = \frac{-x}{(-x)^2-9}$

$= \frac{-x}{x^2-9}$

$= -f(x)$

$\therefore$ odd fn.

ii) At $x=0$ $f(x)=0$. — 1 – correct only

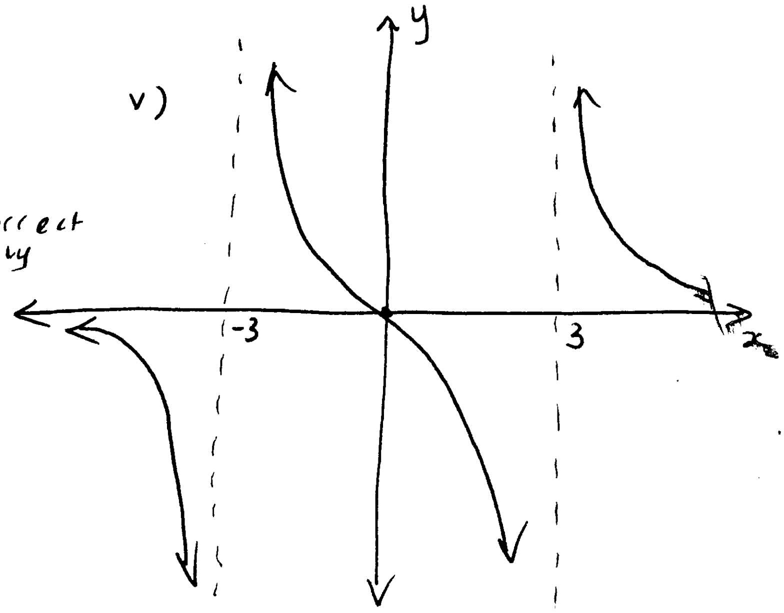
iii) $x = \pm 3$. — 1 – correct only

iv) $\lim\limits_{x \to \infty} \frac{x/x^2}{1-9/x^2}$

$= 0$

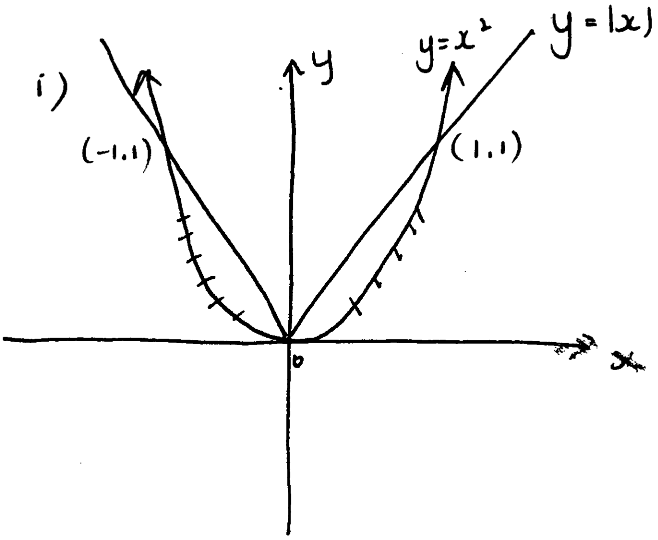
1 – division
1 – answer

v)



*take 1 mark off for each feature not shown!

(c) i)



1 – correct only.

ii) intersect at $(-1,1)$ and $(1,1)$

For $x^2 < |x|$

$-1 < x < 0$ and $0 < x < 1$

1 – point of intersections

1 – must have two separate inequalities.

Question 5

(a) i) $R = \sqrt{a^2 + b^2}$

$= \sqrt{9 + 3}$

$= \sqrt{12}$

$= 2\sqrt{3}.$

$\tan\alpha = b/a$

$= \sqrt{3}/3$

$\alpha = 30°$

$\therefore 3\cos x - \sqrt{3}\sin x = 2\sqrt{3}\cos(x + 30)$

1 – correct only

ii) $\therefore 2\sqrt{3}\cos(x + 30) = -\sqrt{3}$

$\cos(x + 30) = -\frac{1}{2}$

$x + 30 = 120°, 240°$

$x = 90°, 210°$

1 – trig eqn

1 – all solutions

(b) i) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$a \cdot \sin B = b \cdot \sin A$

$\therefore \dfrac{\sin A}{\sin B} = \dfrac{a}{b}$

but $3a = 4b$

$\dfrac{a}{b} = \dfrac{4}{3}$

$\therefore \dfrac{\sin A}{\sin B} = \dfrac{4}{3}$

1 – sine rule equals $\frac{a}{b}$

1 – equate $\frac{a}{b}$ from $3a=4b$.

ii) $\dfrac{\sin 2B}{\sin B} = \dfrac{4}{3}$

$\dfrac{2\sin B\cos B}{\sin B} = \dfrac{4}{3}$

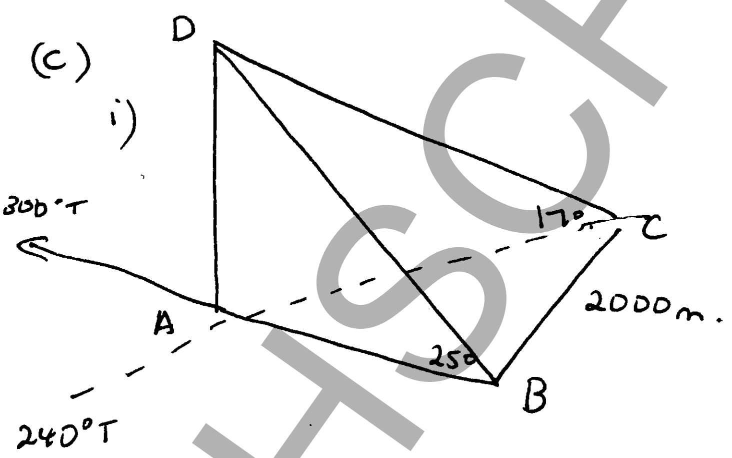
$2\cos B = \dfrac{4}{3}$

$\cos B = \dfrac{2}{3}$

1 – substitution of A with 2B

1 – final answer.

(c) i)



1 – correct only

ii) $\angle BAC = 60°$

1 – correct only

iii) $\tan 25 = \dfrac{h}{AB}$ $\tan 17 = \dfrac{h}{AC}$

$AB = h\cot 25$ $AC = h\cot 17$

In ΔCAB:

1 - expressions for AC, AB

$$2000^2 = AC^2 + AB^2 - 2AC \cdot AB \cos 60$$
$$= h^2\cot^2 25 + h^2\cot^2 17 - 2h\cot 25 \cdot h\cot 17 \cdot \cos 60$$

1 - cos rule

$$= h^2(\cot^2 25 + \cot^2 17 - 2\cot 25 \cot 17 \cos 60)$$
$$h^2 = \frac{2000^2}{\cot^2 25 + \cot^2 17 - 2\cot 25 \cot 17 \cos 60}$$

1 - rearranging

$$h = \frac{2000}{\sqrt{\cot^2 25 + \cot^2 17 - 2\cot 25 \cot 17 \cos 60}}$$

<u>Question 6</u>

(a) $m_1 = 1$

$$\frac{dy}{dx} = 3x^2$$

At $x = -1$

$dy/dx = 3$

$\therefore m_2 = 3$

1 - both gradients correct

1 - substitution into formula

1 - answer

$$\therefore \tan\theta = \left|\frac{3-1}{1+3\times 1}\right| = \left|\frac{2}{4}\right| = \frac{1}{2}$$

$\theta = 26°34'$ (nearest minute)

or

$27°$.

(b) $y = 2x^2 + 2x$

$y' = 4x + 2$

At $x = 3$ $y' = 14$

$\therefore y - 24 = 14(x - 3)$

$y - 24 = 14x - 42$

$y = 14x - 18$

1 - derivative

1 - gradient

1 - eqn.

(c) $f(x) = 4x(3x^2+7)^5$

$u = 4x \quad v = (3x^2+7)^5$

$u' = 4 \quad v' = 5(3x^2+7)^4 \cdot 6x$

$= 30x(3x^2+7)^4$

$\therefore f'(x) = uv' + vu'$

$= 4x \cdot 30x(3x^2+7)^4 + 4 \cdot (3x^2+7)^5$

$= 4(3x^2+7)^4[30x^2 + 3x^2+7]$

$= 4(3x^2+7)^4(33x^2+7)$

1 – correct u', v'

1 – correct substitution

1 – factorisation.

(d) m of tangent $= -\frac{1}{2}$

$y = \frac{x^2}{12} - \frac{px}{12} + \frac{4}{12}$

$\therefore y' = \frac{2x}{12} - p/12$

$= x/6 - p/12$

At $x=1$ $y' = -\frac{1}{2}$

$\therefore -\frac{1}{2} = 1/6 - p/12$

$-2/3 = -p/12$

$p = 8$

1 – derivative

1 – equate derivative with gradiant of tangent

1 – value of p.

Question 7

(a) $3^{2x} + 26 \cdot 3^{x} \times 3^{-1} = 3$

$3^{2x} + \frac{26}{3} \cdot 3^{x} - 3 = 0$

Let $u = 3^{x}$

$u^2 + \frac{26}{3}u - 3 = 0$

$3u^2 + 26u - 9 = 0.$

$\frac{(3u-1)(3u+27)}{3} = 0$ $\quad \times -27$ $\quad + 26$

1 – correct quad

1 – answers

$(3u-1)(u+9) = 0$

$u = \frac{1}{3}$ or $u = -9$

ie. $3^{x} = \frac{1}{3}$ or $3^{x} = -9$

$x = -1$ $\quad$ no solns.

(b) $\Delta = 0$.

$dy = -cx - e$ $\qquad y = \frac{x^2}{4a}$

$y = -\frac{c}{d}x - \frac{e}{d}$

$\therefore \frac{-cx-e}{d} = \frac{x^2}{4a}$

1 – solving simultaneous eqns

$x^2 d = 4a(-cx-e)$

$x^2 d = -4acx - 4ae$

1 – discriminant

$x^2 d + 4acx + 4ae = 0$

1 – let discriminant =0 and simplify

$\Delta = b^2 - 4ac$

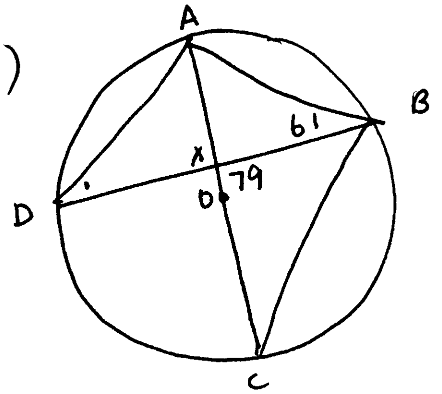
$= 16a^2c^2 - 4(d)(4ae)$

$\therefore 0 = 16a^2c^2 - 16ade$

$\cancel{16}a^2c^2 = \cancel{16}ade$

$ac^2 = de$

(c)



$\angle ABC = 90°$ ($\angle$ in semicircle is right $\angle$)

$\therefore \angle XBC = 29°$

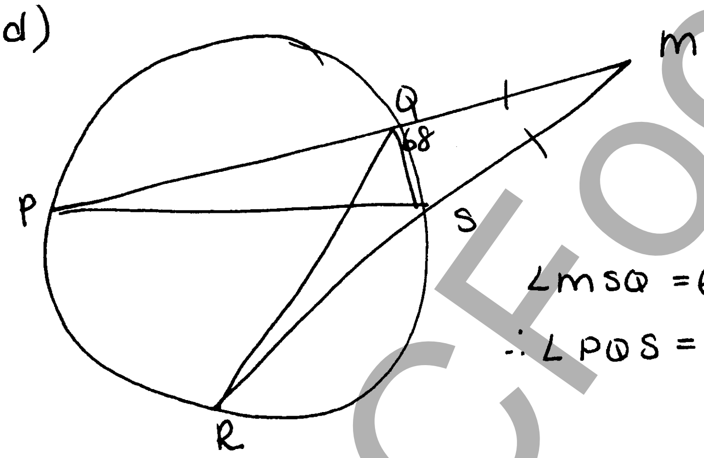
$\therefore \angle XCB = 180 - 29 - 79$ ($\angle$ sum ΔXBC)

$= 72°$

$\therefore \angle ADX = 72°$ ($\angle$ at circum. are equal when standing on same arc)

(-1) mk for each incorrect step in solution.

(d)



$\angle MSQ = 68°$ (isos Δ, base $\angle$)

$\therefore \angle PQS = \angle RSQ = 112°$ (straight $\angle$)

In ΔPQS and ΔQRS:

QS common side

$\angle PQS = \angle RSQ$ (above)

$\angle QPS = \angle QRS$ ($\angle$ at circum are equal from same chord)

$\therefore \Delta PQS \equiv \Delta QRS$ (AAS)

$\therefore QP = SR$ (corres sides cong Δ)

and $MQ = MS$ (given)

$\therefore QP + MQ = SR + SM$

$\therefore MP = MR$

(-1) mk for any incorrect step in solution.

* alt methods acceptable!