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MANLY

IN OMNIBUS GLORIFICETUR DEUS

2005
HALF YEARLY
EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 90 minutes
- Write using a blue or black pen
- Approved calculators may be used
- All necessary working should be shown for every question.
- **Write your Name and Teacher's Name on each piece of paper**
- Begin each question on a fresh sheet of paper.

Total marks (**75**)

- Attempt Questions 1-4

Total Marks (75)
Attempt questions 1-4

Answer each question on a separate piece of paper. Extra paper is available if needed.

Question 1 (16 marks)

a. (i) Write 0.0081 as a simple fraction. **1**

(ii) Simplify $(0.0081)^{-\frac{3}{4}}$ leaving your answer in simplest rational form. **2**

b. Find integers a and b such that $\frac{1}{1-\sqrt{2}} = a - \sqrt{b}$. **3**

c. Factorise fully: $80x^4 - 5y^4$ **3**

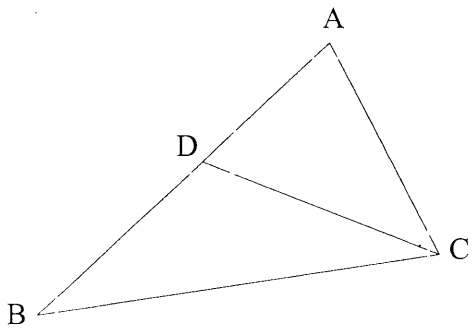
d. Solve for x if: (a) $64^x = \sqrt{32}$. **2**

(b) $(x-1)(x+3) \geq 0$ **2**

(c) $\frac{3}{x+5} \geq 2$. **3**

Question 2 (9 marks)

a.

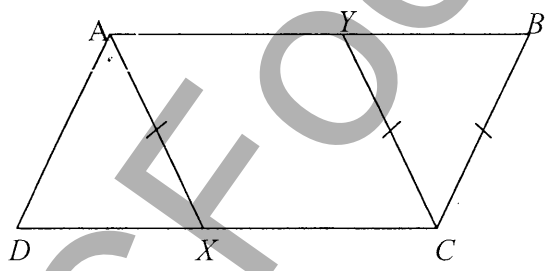


(i) Given $\angle ABC = \angle ACD$, show that $\Delta ABC \parallel \Delta ACD$. **2**

(ii) Hence show that $AC^2 = AB \times AD$. **2**

(iii) Find the length of AB if AC=6cm, AD=4cm. **1**

b.



$ABCD$ is a parallelogram.
The point X lies on CD and Y lies on AB so that $AX = CY = BC$.
Copy the diagram into your answer booklet.

(i) Explain why $\angle ADX = \angle CBY$. **1**

(ii) Show that $AD = AX$. **1**

(iii) Show that $\angle DAX = \angle BCY$. **2**

Question 3 (28 marks)

a. Write down the equation of the circle with centre at the origin and passing through the point (5,7). **2**

b. On separate number planes, draw neat sketches, showing important features, of the following:

(i) $x^2 + y^2 = 5$ **2**

(ii) $y = \dfrac{1}{x+2}$ **2**

c. State the largest possible domain for the function $f(x) = \dfrac{3}{x^2 - 4}$ **2**

d. What is the minimum value of $f(x) = x^2 - 10x + 21$? **2**

e. Find the domain and range of:

$g(x) = \sqrt{x+5}$ **2**

f. Determine (with reasons) whether the following function is odd, even or neither:

$z(x) = \dfrac{7x}{x^4 - 2}$ **2**

g. Draw neat sketches of the following showing the important features:

(i) $y = 5^x - 1$ **2**

(ii) $y = 10x - 4x^2$ **2**

(iii) $y = |2x - 1|$ **2**

h. Shade the region where:

$$x \leq 2, \quad y > -1, \quad y \leq x^2$$ **2**

i. Consider $f(x) = \dfrac{3x^2}{x^2 - 4}$

(i) Show $f(x)$ is an even function. **1**

(ii) Show $f(x)$ has only one intercept with the axes at the origin. **1**

(iii) Determine the equation of the horizontal asymptote. **2**

(iv) Sketch $f(x) = \dfrac{3x^2}{x^2 - 4}$ showing the important features. **2**

Question 4 (22 marks)

a. Give the exact value of $\sin 300^0$ **1**

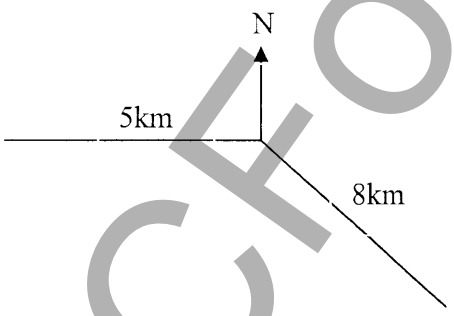
b. Given that $\sin\theta = \frac{3}{\sqrt{15}}$ and $\cos\theta < 0$, find the exact value of $\tan\theta$. **2**

c. Simplify $\frac{\sin(90^0 - \theta)}{\sin(180^0 - \theta)}$. **2**

d. Solve for θ: (i) $\sin 2\theta = \frac{\sqrt{3}}{2}$ for $0^0 \le \theta \le 360^0$ **3**

(ii) $4\sin^2\theta = 1$ for $-180^0 \le \theta \le 180^0$ **3**

e. I walk 5km due east, then 8km on a bearing of 130°. Use the Cosine Rule to find the straight line distance between my starting point and finishing point. **2**



f. A ship sails from port A, 30km due East to port B. It then sails a further 18km in the direction 155^0T to port C.

(i) Draw a neat sketch of the above information. **1**

(ii) Show that $\angle ABC = 155^0$. **1**

(iii) Find the distance from port A to port C. **2**

g. Prove the following identities.

(i) $\dfrac{1+\tan^2 x}{1+\cot^2 x} = \tan^2 x$ **2**

(ii) $\tan A \sin A + \cos A = \sec A$ **3**

END OF EXAMINATION