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Name: ....

Teacher: ....

2011

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- All necessary working should be shown in every question
- Start each question in a new booklet
- Marks may be deducted for careless or untidy work

Total Marks – 84

- Attempt Questions 1-7
- All questions are of equal value

Mark	**/84**
Rank	**/**
Highest Mark	**/84**

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Question 1. (12 Marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Find $\int \frac{2}{\sqrt{36-x^2}}\,dx$. **1**

(b) Differentiate $\tan^{-1}(\ln x)$ with respect to x. **1**

(c) Find the coordinates of the point P that divides the interval joining A (-3, 8) and B (7, -3) internally in the ratio 2 : 3. **2**

(d) Solve $\frac{4}{x-2} \leq 2$. **3**

(e) The acute angle between the lines $2x - y = 4$ and $y = mx + 3$ is $45°$. Find the two possible values of m. **2**

(f) Use the substitution $u = x - 3$ to evaluate $\int_4^5 \frac{x}{\sqrt{x-3}}\,dx$. **3**

Question 2 (12 Marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Let $f(x) = 3\sin^{-1} 2x$. **2**

Sketch the graph of $y = f(x)$, clearly indicating the endpoints for the domain and the range.

(b) (i) Differentiate $x\cos^2 x$ with respect to x. **2**

(ii) Hence, or otherwise, find $\int x \sin 2x \, dx$. **2**

(c) The polynomial $P(x) = x^3 + ax^2 - 2x + b$ has $(x+1)$ as a factor. $P(x)$ has a remainder of 4 when divided by $(x-3)$. **3**

Find the values of a and b.

(d) The function $f(x) = x - e^{-2x}$ has one root between $x = 0$ and $x = 1$. **3**
Use one application of Newtons' method, starting at $x = 0.3$, to find another approximation for this root.

Write your answer correct to 2 decimal places.

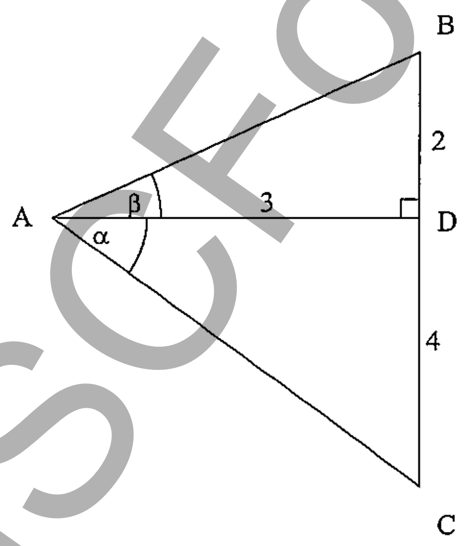
Question 3 (12 Marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Evaluate $\lim\limits_{x\to\infty} \dfrac{3x^2+1}{2-7x-5x^2}$. **2**

(b) (i) Express $\sqrt{3}\cos\theta - \sin\theta$ in the form $r\cos(\theta+\alpha)$, where $r > 0$, and $0 < \alpha < \dfrac{\pi}{2}$, giving r and α as exact values. **2**

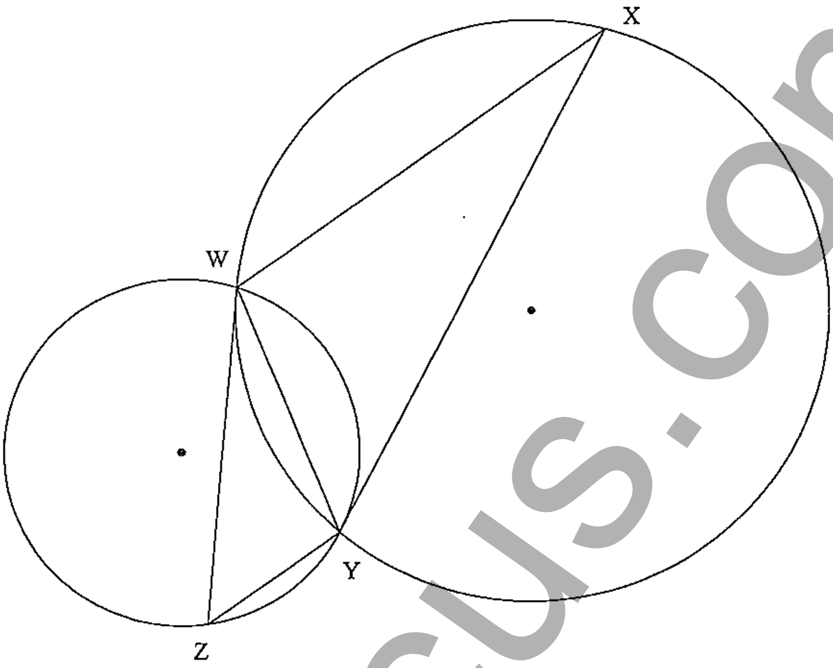
(ii) Solve $\sqrt{3}\cos\theta - \sin\theta = -1$, for $0 \le \theta \le 2\pi$, giving your answers as exact values. **2**

(c) In the diagram below, AD is perpendicular to BC. **3**
CD = 4, BD = 2 and AD = 3. $\angle CAD = \alpha$ and $\angle BAD = \beta$.



Find the exact value of $\sin(\alpha - \beta)$.

(d) WZ and XY are tangents to the circles WXY and WYZ respectively. The circles share two common points, W and Y. **3**
Copy or trace the diagram into your Writing Booklet.



Prove that WX || YZ.

Question 4 (12 Marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) Find the exact value of $\int_0^{\ln 3} \frac{e^x}{e^x + 9} dx$. **2**

(b) Find the constant term in the expansion $\left(2x - \frac{1}{x}\right)^6$. **2**

(c) Prove by induction that $4p + 3p^2 + 2p^3$ is divisible by 3 for $p = 1, 2, 3 ...$ **3**

(d) The temperature (T°C) of steel, after it has been removed from a hot furnace, after t minutes, satisfies the differential equation:

$$\frac{dT}{dt} = k(T - 22) \quad \text{where } k \text{ is a constant.}$$

Initially, the temperature (T) of the steel is 100°C and when t=15minutes, T=70°C.

(i) Use this information to find the exact values of A and k. **2**

(ii) Hence find the value of t when T = 40°C to the nearest minute. **1**

(e) Show that: **2**

$$\sqrt{\frac{1+\sin 2\theta}{1-\sin 2\theta}} = \frac{1+\tan\theta}{1-\tan\theta}.$$

Question 5 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) The speed V cm/s of a particle moving along the x – axis is given by

$$V^2 = 27 + 18x - 9x^2 \text{ where } x \text{ is in cm.}$$

(i) Prove that the motion is Simple Harmonic. Find the period and amplitude of the motion. **3**

(ii) Find the acceleration of the particle when it is 1cm away from the centre of motion. **1**

(b) A stone is projected upwards from the edge of a cliff with a speed of 30m/s. It hits an object 120 m horizontally from the edge and 35 m vertically below it.

Assume that t seconds after the release, the position of the stone is given by

$$x = 30t\cos\alpha \quad \text{and} \quad y = -5t^2 + 30t\sin\alpha \,.$$

(i) Find α, the angle of projection, to the nearest minute. **3**

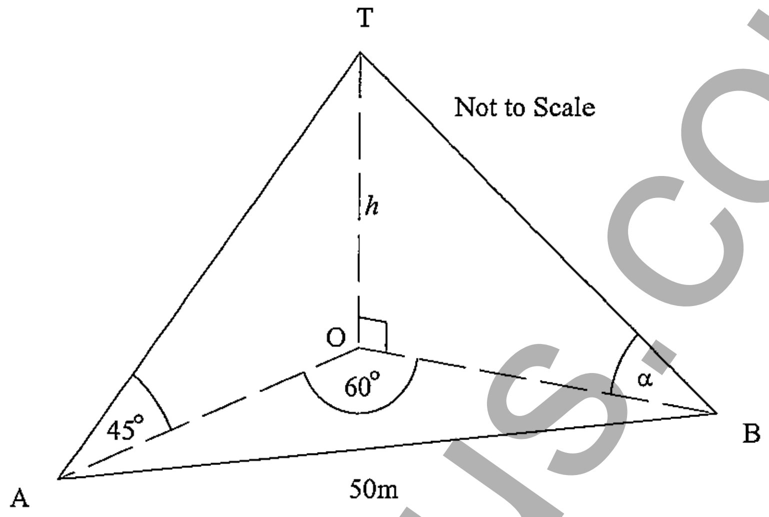
(ii) Find the time taken for the stone to hit the object. **1**

(c) (i) Sketch the curve $y = x + \dfrac{4}{x}$ showing clearly all stationary points and asymptotes. **3**

(ii) Hence, find the values of k such that $x + \dfrac{4}{x} = k$ has no real roots. **1**

Question 6 (12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a)



In the above diagram, the points A, B and O are in the same horizontal plane. A and B are 50 m apart and $\angle AOB = 60^\circ$. OT is a vertical tower of height h metres.

The angles of elevation of T from A and B respectively are 45° and α° (α is acute).

(i) Explain why AO = h. **1**

(ii) Prove $h^2 \cot^2 \alpha - h^2 \cot \alpha + h^2 = 50^2$. **2**

(iii) Given that the tower is 30m high, find the angle α correct to the nearest degree. **3**

(b) (i) Verify that $\frac{d}{dx}\left(x\sin^{-1}x+\sqrt{1-x^2}\right)=\sin^{-1}x.$ **1**

(ii) Hence, using a similar expression, find a primitive of $\cos^{-1}x$. **1**

(iii) The curves $y=\sin^{-1}x$ and $\cos^{-1}x$ intersect at P. **1**

The curve $y=\cos^{-1}x$ also intersects the x axis at Q.

Show that P has co-ordinates $\left(\frac{1}{\sqrt{2}},\ \frac{\pi}{4}\right)$.

(iv) Find the area enclosed by the x – axis and the arcs OP and PQ. **3**

Question 7(12 marks) Use a **SEPARATE** Writing Booklet. **Marks**

(a) A particle is moving in a straight line with acceleration given by

$$\frac{d^2x}{dt^2} = 9(x-2).$$

where x is the displacement in metres, from the origin O after t seconds. Initially the particle is 4m to the right of O and it has a velocity of $V = 6$m/s.

(i) Show that $V^2 = 9(x-2)^2$. **3**

(ii) Find an expression for V and hence find x as a function of t. **3**

(b) PQ is a variable chord of the parabola $x^2 = 4ay$ which subtends a right angle at the vertex.

(i) If p and q are the parameters corresponding to the points P and Q, prove that $pq = -4$. **2**

(ii) Show that the equation of the normal at P is

$$x + py = 2ap + ap^3.$$ **2**

(iii) Hence prove that the locus of the point of intersection of the normals at P and Q is the parabola $x^2 = 16a(y-6a)$. **2**

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$