

START A NEW PAGEQUESTION ONE (12 marks)

a) Evaluate

$$\int_0^3 \frac{dx}{x^2+9}.$$

2

b) Find the acute angle between lines  $2x - y = 0$  and  $x + 3y = 0$ .

3

Give your answer correct to the nearest minute.

c) Solve  $\frac{x}{2x-1} \leq 5$ .

3

d) Differentiate  $\cos^4 x$  with respect to  $x$  and hence find  $\int \sin x \cos^3 x dx$ .

2

e)  $A$  is the point  $(-2, -1)$  and  $B$  is the point  $(1, 5)$ .

2

Find the coordinates of  $Q$  which divides  $AB$  internally in the ratio 5:3.START A NEW PAGEQUESTION TWO (12 marks)a) Evaluate  $\int_0^{\frac{\pi}{3}} \sin^2(2x) dx$ .

3

b) Use the substitution  $u = \tan x$ , to evaluate  $\int_0^{\frac{\pi}{3}} \tan^2 x \sec^2 x dx$ .

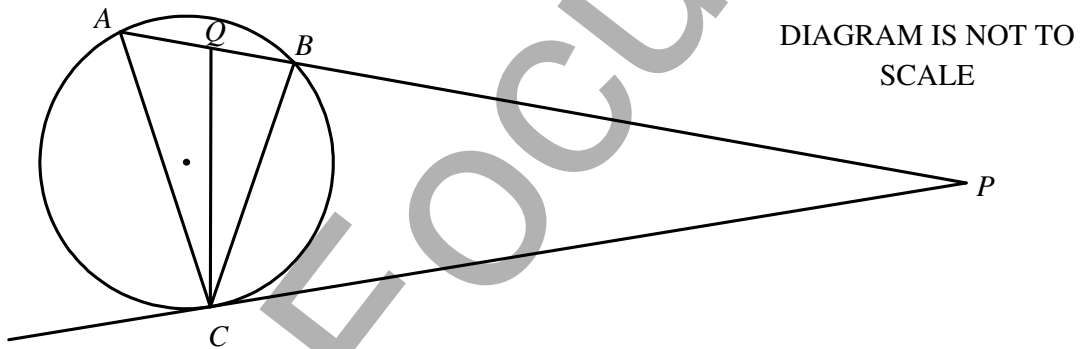
3

- c) A biased coin has a probability of 0.6 of showing a head when it is tossed.  
The biased coin is tossed five times.
- (i) What is the probability of tossing exactly three heads and two tails? 2
- (ii) What is the probability of getting less than three heads? 2
- (iii) What is the most likely outcome and what is the probability that this event will occur? 2

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QUESTION THREE (12 marks)

- a) Find the exact value of  $\sin\left(\sin^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{-5}{12}\right)\right)$ . 3
- b) In the diagram below  $PC$  is a tangent to the circle at  $C$  and  $QC$  bisects  $\angle ACB$ .



- (i) Copy the diagram into your answer booklet and prove that  $PC = PQ$ . 2
- (ii) If  $PC = 10\text{cm}$  and  $AQ = 2\text{cm}$ , Find the length of  $QB$ . 2

- c) Molten plastic at a temperature of  $250^{\circ}\text{C}$ , is poured into a mould to form a car part. After 20 minutes the plastic has cooled to  $150^{\circ}\text{C}$ . If the temperature after  $t$  minutes, is  $T^{\circ}\text{C}$ , and the surrounding temperature is  $30^{\circ}\text{C}$ , then the rate of cooling is given by:

$$\frac{dT}{dt} = -k(T - 30), \text{ where } k \text{ is constant.}$$

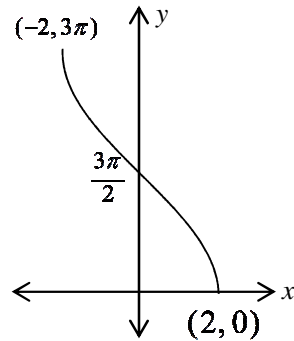
- (i) Show that  $T = 30 + Ae^{-kt}$ , where  $A$  is a constant, satisfies this equation. 1
- (ii) Show that the value of  $A$  is  $220^{\circ}\text{C}$ . 1
- (iii) Find the value of  $k$  to 2 decimal places. 1
- (iv) The plastic can be taken out of the mould when the temperature drops below  $80^{\circ}\text{C}$ . How long after the plastic has been poured will this temperature be reached? Give your answer to the nearest minute. 2

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QUESTION FOUR (12 marks)

- a) Prove by mathematical induction that:  
 $5^n - 4n - 1 \geq 0$ , for all  $n \geq 1$ . 4
- b) John contributes \$800 every quarter into a superannuation fund earning 6% pa, compounded quarterly. Contributions are made for 10 years and then John stops making contributions for 3 years whilst unemployed. He is re-employed, and contributions resume under the previous conditions. John works for a further 7 years until retirement.
- (i) Show that on retirement the contributions for the first 10 years can be expressed as: 1  
 $800(1.015)^{80} + 800(1.015)^{79} + \dots + 800(1.015)^{41}$ .
- (ii) Hence, calculate the total amount in the superannuation fund on retirement. 3

c) Below is the graph  $y = a \cos^{-1} bx$ .



- (i) Show that the value of  $a$  is 3 and that  $b$  is  $\frac{1}{2}$ . 2
- (ii) Find the exact value of the area enclosed by the curve  $y = a \cos^{-1} bx$ , the  $x$  axis and the line  $y = \frac{\pi}{2}$ . 2

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QUESTION FIVE (12 marks)

- a) A particle moves in a straight line so that its acceleration,  $a$ , is given by  $a = 4x$ . The displacement,  $x$ , of the particle is initially 1 metre to the right of the origin with a velocity of  $-2ms^{-1}$ .
- (i) Show that  $v = -2x ms^{-1}$ . 2
- (ii) Express  $x$  as a function of  $t$ . 2
- (iii) Hence, find the displacement when  $t = 2$  seconds to 3 decimal places. 1

- b) (i) Write an expression for  $(1+x)^{20} + (1-x)^{20}$ . 2  
Leave your answer in  ${}^n C_k$  notation.

(ii) Show that  $\sum_{k=0}^{10} {}^{20} C_{2k} x^{2k} = \frac{1}{2} [(1+x)^{20} + (1-x)^{20}]$ . 1

- (iii) Integrate both sides of the equation in part (ii). 2

- (iv) Hence, or otherwise show:

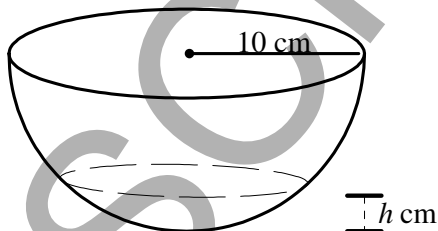
$${}^{20} C_0 + \frac{1}{3} {}^{20} C_2 + \frac{1}{5} {}^{20} C_4 + \dots + \frac{1}{21} {}^{20} C_{20} = \frac{2^{20}}{21}. \quad 2$$

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QUESTION SIX (12 marks)

- a) (i) The diagram below represents a hemispherical bowl of radius 10 cm. It is filled with water to a depth of  $h$  cm. By finding the volume generated by rotating  $x^2 + y^2 = 100$  between  $y=10$  and  $y=10-h$  about the  $y$  axis, show that the volume of water in the bowl is given by:

$$V = \frac{\pi h^2}{3} (30-h) \quad 2$$



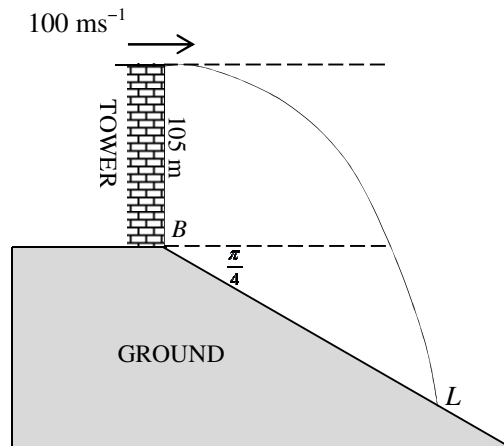
- (ii) The hemispherical bowl referred to above is being filled with water at a constant rate of  $2\pi \text{ cm}^3 \text{ min}^{-1}$ . Find the rate of increase of the depth of the water when the depth is 2 cm. 3

- b) Consider the function  $f(x) = \sqrt{x+3}$
- (i) Find an expression for  $y = f^{-1}(x)$ . 1
- (ii) State the domain for  $y = f^{-1}(x)$ . 1
- (iii)  $f(x)$  and  $f^{-1}(x)$  intersect at exactly one point  $P$ . Let  $\alpha$  be the  $x$  coordinate of  $P$ . 2  
 Explain why  $\alpha$  is a root of the equation  $x - \sqrt{x+3} = 0$ . 2
- (iv) Show that the value of  $\alpha$  occurs when  $2 < \alpha < 3$ . 1
- (v) Take  $x = 2.5$  as the first approximation of  $\alpha$ , use one application of Newton's method to find a second approximation for  $\alpha$  correct to 3 decimal places. 2

START A NEW PAGEQUESTION SEVEN (12 marks)

- a) On a certain day the depth of water in a harbour is 7 metres at low tide (9:25am) and  $10\frac{2}{3}$  metres at high tide (3:40pm). Assume the rise and fall of the surface of the water to be in simple harmonic motion in the form  $\ddot{x} = -n^2(x - b)$  where  $x = b$  is the centre of motion and  $x = a$  is the amplitude. 1
- (i) Show that  $x = b - a \cos nt$  satisfies  $\ddot{x} = -n^2(x - b)$ . 1
- (ii) Find the values of  $a$ ,  $b$  and  $n$ . 3
- (iii) Hence, find the earliest time before 3:40pm on this day, a boat may safely enter the harbour if the minimum depth of  $9\frac{1}{2}$  metres of water is required. 2

- b) A bullet is fired horizontally with a velocity of  $100\text{ms}^{-1}$  from the top of a tower 105 metres high. The tower is at the top of a hill, which slopes downwards at an angle of depression of  $\frac{\pi}{4}$ . The bullet lands at  $L$ , on the ground as represented on the diagram below.



- (i) Consider  $B$ , the base of the tower, as the origin, and using acceleration due to gravity as  $10\text{ ms}^{-2}$ , show that the expressions for the  $x$  and  $y$  coordinates of the position of the bullet at time  $t$  seconds are:

$$x = 100t \text{ and } y = 105 - 5t^2.$$

- (ii) Show that the equation of the line  $BL$  is  $y = -x$ .
- (iii) Find the time taken for the bullet to hit the ground at  $L$ .
- (iv) Find the distance  $BL$  to the nearest metre.

END OF THE EXAMINATION