Question 1 (12 marks) Begin a new booklet **Marks**

(a) Factorise fully $16x^3 - 2$ **2**

(b) Consider the points $A(-3, 2)$ and $B(6, -4)$. Find the coordinates of the point $P(x, y)$ that divides the interval AB in the ratio of 2:1. **2**

(c) Solve the inequality $\frac{2}{x+1} < 1$. **2**

(d) Consider $f(x) = 3x^3 + 6$. Explain, using calculus, why $f(x)$ is always increasing. **2**

(e) Differentiate:

(i) $x\cos^2 3x$ **2**

(ii) $x^2 \tan^{-1} x$ **2**

Question 2 (12 marks) Begin a new booklet **Marks**

(a) Find the values of k such that $(x-2)$ is a factor of the polynomial **2**
$P(x)=x^3-2x^2+kx+k^2$.

(b) Find correct to the nearest degree the acute angle between the lines **2**
$y=3x+1$ and $x+y-5=0$

(c) Evaluate $\int_0^1 \frac{dx}{\sqrt{4-x^2}}$ **2**

(d) Use the substitution $t=\tan\frac{x}{2}$ to show that $\frac{1-\cos x}{\sin x}=\tan\frac{x}{2}$ **3**

(e) Graph $y=2\sin 3x$ for $0 \le x \le 2\pi$ **3**

Question 3 (12 marks) Begin a new booklet **Marks**

(a) If $5x^3 - 6x^2 - 29x + 6 = 0$ has roots α, β, γ then find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$ **2**

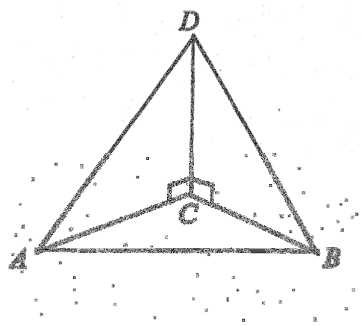
(b) Use Mathematical Induction to prove that $2^n \geq 1+n$ for $n \geq 1$ **4**

(c) Use the substitution $u = x^2 + 1$ to evaluate $\int_1^7 \frac{x}{(1+x^2)^2}dx$. **4**

(d) Find the sum of the infinite series $\sin^2 x + \sin^4 x + \sin^6 x + \ldots$ Express your answer in simplest form. **2**

Question 4 (12 marks) Begin a new booklet **Marks**

(a)



Three points *A, B* and *C* lie on a plane. Points *A* and *B* are 30 metres apart and $\angle ACB = 120^\circ$. A vertical flagpole, *CD*, of height *h* metres stands at *C*. From *A* the angle of elevation of the top, *D,* of the flagpole is 30°. From *B* the angle of elevation to *D* is 45°

(i) Find the length of AC and BC in terms of *h*. **2**

(ii) Hence find the value of *h* to one decimal place. **2**

(b) The parametric equations of a parabola are

$$x = 2t$$
$$y = 2t^2$$

(i) Find the Cartesian equation of the parabola. **1**

(ii) Find the equation of the tangent to this parabola at the point where $x = 2$. **2**

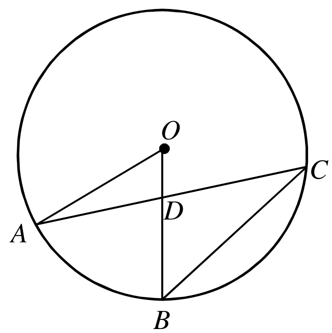
(c) Find $\int \sin^2 2x \, dx$ **2**

(d) (i) Find the domain and range of the function $f(x) = \cos^{-1}(2x)$. **2**

(ii) Sketch the graph of the curve $f(x) = \cos^{-1}(2x)$. **1**

Question 5 (12 marks) Begin a new booklet **Marks**

(a)



A, B and C are points lying on circle with centre O. AO is parallel to BC. OB and AC intersect at D. $\angle ACB = 31^\circ$.

(i) Find the size of $\angle AOB$, giving reasons. **2**

(ii) Find the size of $\angle BDC$, giving reasons. **2**

(b) At time t years after the start of the year 2000, the number of individuals in A population is given by $N = 80 + Ae^{0.1t}$ for some constant $A > 0$.

(i) Show that $\dfrac{dN}{dt} = 0.1(N - 80)$. **1**

(ii) If there were 100 individuals in the population at the start of the year 2000 Find the year in which the population size is expected to reach 200. **3**

(c) (i) Write $\sqrt{3}\cos x - \sin x$ in the form $r\cos(x + \alpha)$. **1**

(ii) Hence, give the general solution to $\sqrt{3}\cos x - \sin x = 1$ **3**

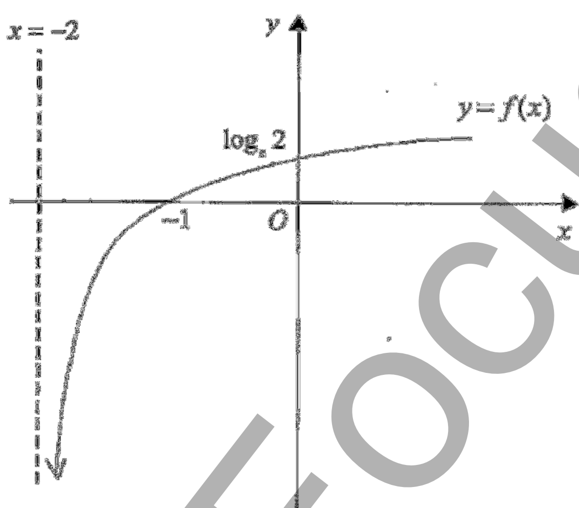
Question 6 (12 marks) Begin a new booklet **Marks**

(a) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O in the line, v ms^{-1} is given by $v = \dfrac{1}{x+1}$ and acceleration a ms^{-2}. Initially the particle is at O.

(i) Express a as a function of x. **1**

(ii) Express x as a function of t. **3**

(b)



The diagram shows the graph of the function $f(x) = \ln(x+2)$.

(i) Copy the diagram and on it draw the graph of the inverse function $f^{-1}(x)$ showing the intercepts on the axes and the equation of the asymptote. **2**

(ii) Show that the x-coordinates of the points of intersection of the curves $y = f(x)$ and $y = f^{-1}(x)$ satisfy the equation $e^x - x - 2 = 0$. **2**

(iii) Show that the equation $e^x - x - 2 = 0$ has a root α such that $1 < \alpha < 2$. **2**

(iv) Use one application of Newton's method with an initial approximation $\alpha_0 = 1.2$ to find the next approximation for the value of α, giving your answer correct to one decimal place. **2**

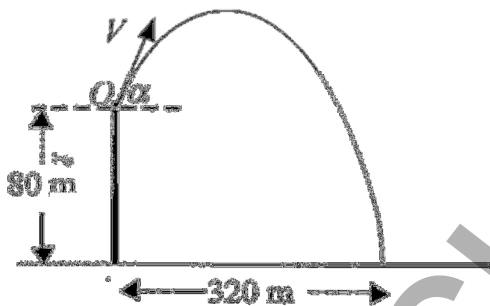
Question 7 (12 marks) Begin a new booklet **Marks**

(a) A particle is moving in a straight line with Simple Harmonic Motion. At time t seconds it has displacement x metres from a fixed point O on the line, given by $x = 1 + 3\cos\frac{t}{2}$, velocity v ms^{-1} and acceleration a ms^{-2}.

(i) Show that $a = -\frac{1}{4}(x-1)$. **2**

(ii) Find the distance travelled and the time taken by the particle over one complete oscillation of its motion. **2**

(c)



A particle is projected with speed V ms^{-1} at an angle α above the horizontal from a point O at the top edge of a vertical cliff which is 80 m above horizontal ground. The particle moves in a vertical plane under gravity where the acceleration due to gravity is $10\,\text{ms}^{-2}$. It reaches its greatest height after 3 seconds and hits the ground at a horizontal distance of 320 m from the foot of the cliff.

The horizontal and vertical displacements, x and y metres respectively, of the particle from the point O after t seconds are given by $x = Vt\cos\alpha$ and $y = -5t^2 + Vt\sin\alpha$. (Do NOT prove these results.)

(i) Show that $V\sin\alpha = 30$. **1**

(ii) Show that the particle hits the ground after 8 seconds. **2**

(iii) Show that $V\cos\alpha = 40$. **1**

(iv) Hence find the exact value of V and the value of α to the nearest minute. **2**

(v) Find the time after projection when the direction of motion of the particle first makes an angle of 45° below the horizontal. **2**