Name:	
Class:	

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2013

MATHEMATICS EXTENSION 1

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 2 hours.
- Write in black or blue pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In every question, show all necessary working
- Marks may not be awarded for careless or badly arranged working.

Total Marks 70

Section I: **10 marks**

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: **60 Marks**

- Attempt Question 11 - 14
- Answer on blank paper unless otherwise instructed. Start a new page for each new question.
- Allow about 1 hour & 45 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

SECTION I MULTIPLE CHOICE (10 marks)

Attempt Question 1 – 10 (1 mark each)
Allow approximately 15 minutes for this section

1. An object is projected with a velocity of 40 ms^{-1} at an angle of $\tan^{-1}\frac{8}{6}$ to the horizontal. If air resistance is neglected and acceleration due to gravity is taken as 10 ms^{-2}, what is the initial vertical component of its velocity?

(A) 32 ms^{-1}

(B) 24 ms^{-1}

(C) $40\tan\frac{8}{6}$ ms^{-1}

(D) $40\sin\frac{8}{10}$ ms^{-1}

2. For the function $f(x) = 3\sin^{-1}(\frac{x}{4})$, the domain and range of $y = f(x)$ (where x and y are real numbers) are:

(A) $-4 \leq x \leq 4\ ; -\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}$

(B) $-\frac{3\pi}{2} \leq x \leq \frac{3\pi}{2}; -4 \leq y \leq 4$

(C) $-3 \leq x \leq 3\ ; -2\pi \leq y \leq 2\pi$

(D) $-2\pi \leq x \leq 2\pi\ ; -3 \leq y \leq 3$

3. A particle moves in a straight line. At time t seconds, where $t \geq 0$, its displacement x metres from the origin and its velocity v are such that $v = 25 + x^2$. If initially the particle is 5m to the right of the origin, then t is equal to:

(A) $\tan^{-1}(\frac{x}{5}) - \frac{\pi}{4}$

(B) $\frac{1}{5}\tan^{-1}(\frac{x}{5}) - \frac{\pi}{20}$

(C) $25x + \frac{x^3}{3}$

(D) $25x + \frac{x^3}{3} + \frac{500}{3}$

4. What are appropriate values of R and θ such that $\sqrt{3}cosx + sinx \equiv Rcos(x+\theta)$

(A) $R = \sqrt{2}$
$\theta = \frac{\pi}{6}$

(B) $R = \sqrt{2}$
$\theta = \frac{11\pi}{6}$

(C) $R = 2$
$\theta = \frac{\pi}{6}$

(D) $R = 2$
$\theta = \frac{11\pi}{6}$

5. If $cos2x < -sinx$ for $0 \le x \le \pi$, then:

(A) This is true for $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$

(B) There are no x values in this domain for which this is true

(C) This is true for all x values within this domain except $x = \frac{\pi}{2}$

(D) This is true for all x values within this domain

6. $(x - a)$ is a factor of the polynomial $P(x)$, where a is an integer.
If $P(x) = x^3 - kx^2 + 2kx - 8$, the values of k for which $P(x)$ has real roots are:

(A) $-6 \le k \le 2$

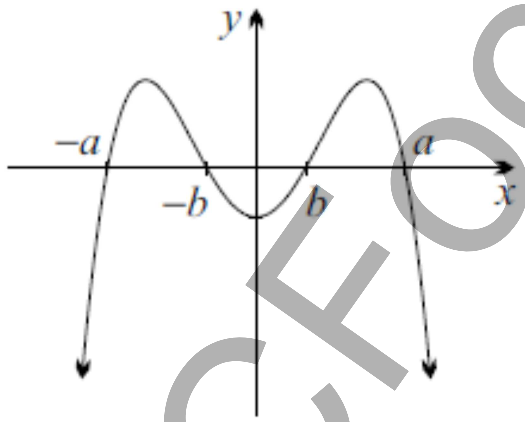
(B) $-2 \le k \le 6$

(C) $k \le -6$ or $k \ge 2$

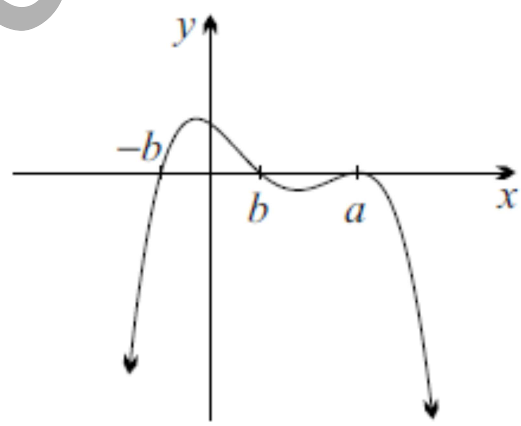
(D) $k \le -2$ or $k \ge 6$

7. Which diagram best represents $y = P(x)$ if $P(x) = (x - a)^2(b^2 - x^2)$, and $a > b$?

(A)



(C)



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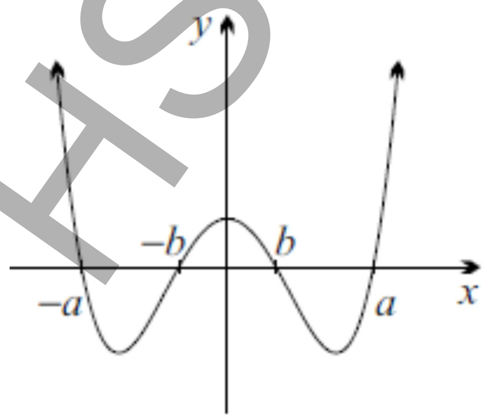
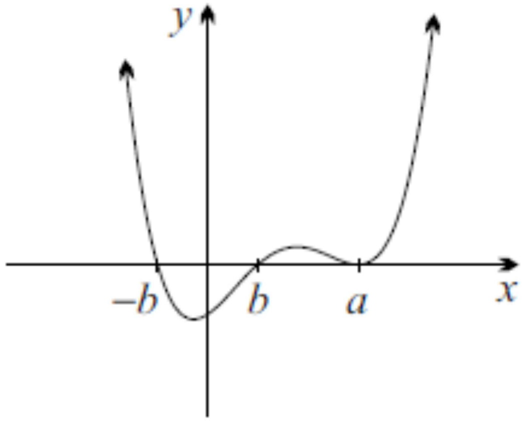


(D)



8. If $cos\theta = -\frac{3}{5}$ and $0 < \theta < \pi$, then $\tan\frac{\theta}{2}$ is equal to:

(A) $-\frac{1}{3}$ or -3

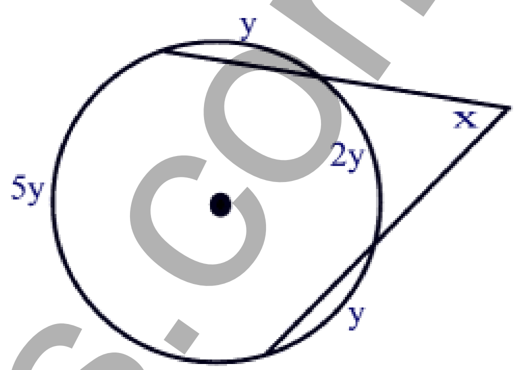
(B) $\frac{1}{3}$ or 3

(C) -2

(D) 2

9. Two secants of equal length intersect at an external point. Given the arc lengths shown in the diagram, the number of degrees in the angle labeled x is:

DIAGRAM NOT TO SCALE



(A) 40

(B) 60

(C) 80

(D) 140

10. Two year 12 students are to be randomly selected from a pool of N year 12 students, n of whom are from James Ruse. If it is known that at least one student is from James Ruse, what is the chance that both students are from James Ruse?

(A) $\frac{n-1}{2N-n-1}$

(B) $\frac{n-1}{2N+n+1}$

(C) $\frac{n-1}{2N+n-1}$

(D) $\frac{n-1}{2N-n+1}$

END OF SECTION I

SECTION II EXTENDED RESPONSE (60 marks)

Total Marks is 60
Attempt Question 11 – 14.
Allow approximately 1 hour & 45 minutes for this section.

*Answer all questions, starting each new question on a new sheet of paper with your **student ID number** in the top right hand corner and the question number on the left hand side of your paper. All necessary working must be shown in each and every question.*

QUESTION 11 *(15 Marks)* START A NEW PAGE **Marks**

(a) The functions $y = 4x^3 + 3x - 1$ and $y = 1 - \ln(2x)$ intersect at $(\frac{1}{2}, 1)$. Find the acute angle between the tangents of the two curves at the point of intersection. Give your answer to the nearest minute. 3

(b) (i) State the range of $y = \tan^{-1}\frac{\sqrt{x^2-4}}{2}$ 1

(ii) Find $\frac{dy}{dx}$ for the function $y = \tan^{-1}\frac{\sqrt{x^2-4}}{2}$ 3

(c) The concentration, C, of good bacteria in a healthy intestine is usually 110 bacteria per gram of intestinal contents. After a bout of food poisoning, this concentration drops to 15 bacteria per gram. The rate of increase of the concentration of good bacteria is proportional to the difference from the normal 110 bacteria per gram, that is,

$$\frac{dC}{dt} = k(110 - C)$$

where t is the number of hours since suffering food poisoning.

(i) Show that $C = 110 - Ae^{-kt}$ satisfies the above equation. 1

(ii) 19 hours after suffering food poisoning, Harold has a concentration of 20 bacteria per gram. How long after suffering food poisoning will the concentration of good bacteria in his intestines reach 90% of its healthy state? Give your answer to the nearest hour. 3

(d) PQ is the chord of contact of the parabola $x^2 = 4y$ from the external point $A(x_1, y_1)$ with equation $xx_1 = 2a(y + y_1)$.

(i) Show that the midpoint, M, of PQ has the coordinates $(x_1, \frac{x_1^2}{2} - y_1)$. 2

(ii) If A moves along the line $3x - y - 1 = 0$, show that the equation of the locus of M is a parabola of the form $(x - h)^2 = \pm 4a(y - k)$. 2

QUESTION 12 *(15 Marks)* START A NEW PAGE **Marks**

(a) Find $\int \frac{x^2}{(a^2-x^2)^{\frac{3}{2}}}\,dx$ by making the substitution $x = a\sin u$. 3

(b) Assume that, for all real numbers x and all positive integers n, $(1+x)^n$ can be written as

$$\sum_{r=0}^{n} {}^{n}C_{r}\, x^{r}$$

Find a simple expression for: 2

$$\sum_{r=0}^{n} r.\ {}^{n}C_{r}$$

(c) The water level at a harbour entrance approximates to simple harmonic motion. On a particular day high tide occurred at 6.00 am. There was a depth of 10 m of water at that time. At low tide, which occurred at 11.50 am, there was a depth of 3 m of water.

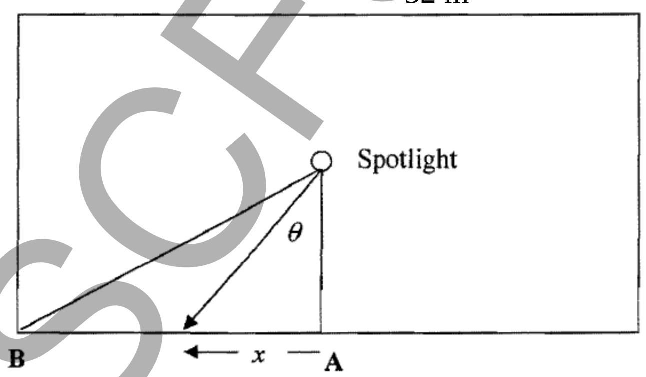
(i) Find, to the nearest cm, the depth of water at the harbour entrance at 2 am earlier that same day. 3

(ii) Write an expression, in exact form, for the general solution for the times, t, when the depth of water at the harbour entrance is 7.5 m. 1

(iii) What is the latest time before midnight that day that a ship can enter the harbour if a minimum depth of 7.5 m of water is required? Give your answer to the nearest minute. 1

(d) A spotlight is in the centre of a rectangular room which measures 32 m by 22m. It is spinning in a clockwise direction at a rate of 25 revolutions/min. Its beam throws a spot which moves along the wall as it spins.

32 m



22 m

Top view
NOT TO SCALE

(i) Write the rate of rotation $\frac{d\theta}{dt}$ in radians/sec. 1

(ii) Find an expression for the velocity $\frac{dx}{dt}$ in terms of x at which the spot appears to be moving along the wall from A to B. 2

(iii) Amy stands at point A while Barry stands at point B. They notice that from their different positions in the room the speed at which the spot appears to be moving is also different. From whose position will the spotlight appear to be moving faster? Justify your answer. 2

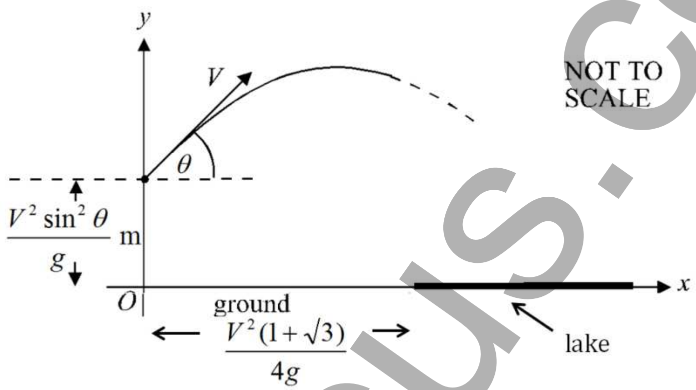
QUESTION 13 *(15 Marks)* START A NEW PAGE **Marks**

(a) The probability of winning a certain game is $\frac{1}{3}$. How many times should the game be played so that the probability of winning 4 times is 60 times the probability of winning 6 times? **3**

(b) (i) A particle moves in a straight line with acceleration $\frac{dv}{dt} = -3 + 9x$. Initially, the particle is at $x = -1$ with a velocity of $4\ ms^{-1}$. Find x as a function of t. **4**

(ii) Describe the motion of the particle when $x = 3$. **1**

(c)



An archer stands at the edge of a cliff and shoots an arrow at a constant velocity of $V\ ms^{-1}$ and at an angle of θ to the horizontal, where $0° < \theta < 90°$. The arrow that he shoots is released from a point $\frac{V^2 \sin^2 \theta}{g}$ m vertically above the ground. At ground level, $\frac{V^2(1+\sqrt{3})}{4g}$ m away horizontally from the point of projection is a lake that is $\frac{V^2}{2g}$ m wide. The position of the arrow at time t seconds after it is projected is given by the equations:

$$x = Vt\cos\theta \ ; \qquad \text{and } y = -\frac{gt^2}{2} + Vt\sin\theta + \frac{V^2 \sin^2 \theta}{g}$$

(i) Show that the Cartesian equation of the path of the arrow is given by **1**

$$y = \frac{-gx^2 \sec^2 \theta}{2V^2} + x\tan\theta + \frac{V^2 \sin^2 \theta}{g}$$

(ii) Show that the horizontal range of the arrow on the ground is given by **2**

$$x = \frac{V^2\left(1 + \sqrt{3}\right)\sin 2\theta}{2g}$$

(iii) Find the values of θ for which the arrow will **not** land in the lake or on the edge of the lake. **4**

QUESTION 14 *(15 Marks)* START A NEW PAGE **Marks**

(a) (i) Explain why for every positive integer $n, n(n+1)$ is even (no formal proof required). 1

(ii) Hence, using the Principle of Mathematical Induction, prove that for every integer $n \geq 2, n^3 - n$ is a multiple of 6. 3

(b) Find the volume of the solid formed when the region enclosed entirely by the curves $y = \sin x$ and $y = \sin 2x$ over the domain $0 \leq x \leq \frac{\pi}{2}$ is rotated about the x-axis. 4

(c) A, B and C are three points on the circumference of a circle. CB is produced to meet the tangent from A at T. If M is the midpoint of BC, prove that $\angle AOT = \angle AMT$. 3

(d) The number of ways of arranging n students in a row such that no two boys sit together and no two girls sit together is m (m > 100). If one more student is added, the number of ways of arranging the students as above increases by 200%.

(i) Explain why the difference between the number of boys and the number of girls cannot be more than 1. 1

(ii) Show that n cannot be odd. 2

(iii) Hence, or otherwise, find the value of n. 1

END OF PAPER