BAULKHAM HILLS HIGH SCHOOL

TRIAL 2013
YEAR 12 TASK 4

Mathematics Extension 1

General Instructions
- Reading time – 5 minutes
- Working time – 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-14
- Marks may be deducted for careless or badly arranged work

Total marks – 70
Exam consists of 10 pages.

This paper consists of TWO sections.

<u>Section 1</u> – Page 2-4 (10 marks)
Questions 1-10
- Attempt Question 1-10

<u>Section II</u> – Pages 5-9 (60 marks)
- Attempt questions 11-14

Table of Standard Integrals is on page 10

Section I - 10 marks
Use the multiple choice answer sheet for question 1-10

1. The value of $\lim_{x \to 0} \frac{\sin 7x}{3x}$ is

(A) $2\frac{1}{3}$

(B) $\frac{3}{7}$

(C) 0

(D) 1

2. What is the acute angle, to the nearest degree, between the lines $y = 7 - 4x$ and $2x - 3y - 6 = 0$.

(A) 26°

(B) 48°

(C) 70°

(D) 75°

3. Let α, β and γ be the roots of $2x^3 + x^2 - 4x + 9 = 0$.

What is the value of $\frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$

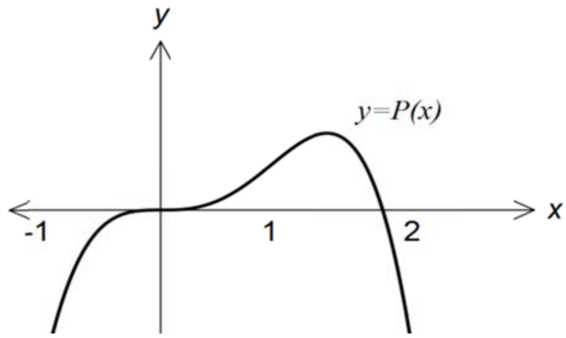
(A) $-\frac{1}{9}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{9}$

(D) $\frac{1}{2}$

4. Which of the following could be the polynomial $y = P(x)$



(A) $P(x) = x^3(2 - x)$

(B) $P(x) = x^2(2 - x)^2$

(C) $P(x) = x^3(x - 2)$

(D) $P(x) = -x^3(x + 2)$

5. The coordinates of the points A and B are $(-1,1)$ and $(3,-1)$ respectively.
Find the coordinates of P that divides the interval AB externally in the ratio $1:3$.

(A) $\left(2, -\frac{1}{2}\right)$

(B) $\left(-1\frac{1}{2}, \frac{1}{2}\right)$

(C) $(-3,2)$

(D) $(4,-2)$

6. The number of different arrangements of the letters of the word **R E G I S T E R** which begin and end with the letter R is:

(A) $\frac{6!}{(2!)^2}$

(B) $\frac{8!}{(2!)}$

(C) $\frac{6!}{2!}$

(D) $\frac{8!}{2!2!}$

7. The inverse function of $g(x)$ where $g(x) = \sqrt{3x - 8}$ is

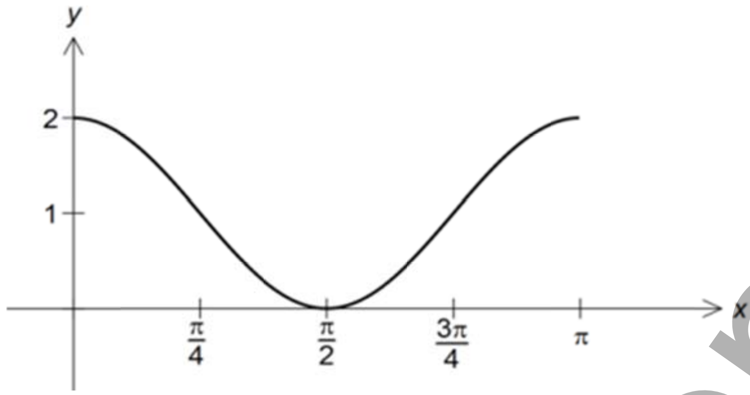
(A) $g^{-1}(x) = \frac{x^2+8}{3}$

(B) $g^{-1}(x) = (3x - 8)^2$

(C) $g^{-1}(x) = \sqrt{\frac{x}{3} + 8}$

(D) $g^{-1}(x) = \frac{x^2-8}{3}$

8. A possible equation for the curve drawn is:-



(A) $y = -1 + \cos 2x$

(B) $y = 1 + \cos 2x$

(C) $y = \cos(2x + 1)$

(D) $y = 1 - \cos 2x$

9. The middle term in the expansion $(2x - 4)^4$ is

(A) 81

(B) $216x^2$

(C) $384x^2$

(D) $-96x^3$

10. If $y = \sin^{-1}(2x)$, then $\frac{d^2y}{dx^2}$ equals

(A) $\frac{-8}{\sqrt{(1-4x^2)^3}}$

(B) $\frac{4x}{\sqrt{1-4x^2}}$

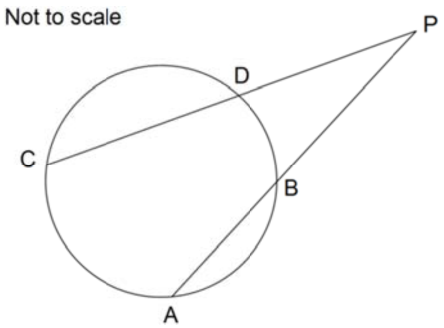
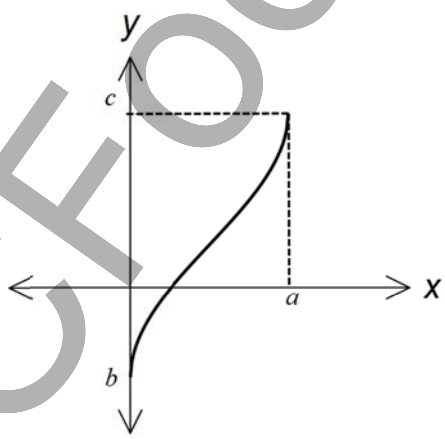
(C) $\frac{-8x}{\sqrt{(1-4x^2)^3}}$

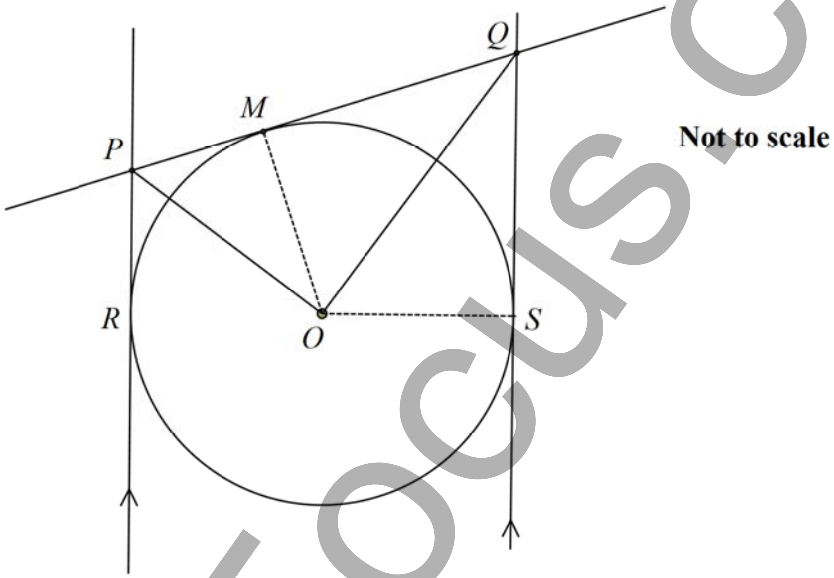
(D) $\frac{8x}{\sqrt{(1-4x^2)^3}}$

End of Section 1

Section II – Extended Response

Attempt questions 11-16.
Answer each question on a SEPARATE PAGE. Clearly indicate question number.
Each piece of paper must show your number.
All necessary working should be shown in every question.

Question 11 (15 marks)		**Marks**
a)	Solve for x $$\frac{2x+1}{1-x} \geq 1$$	**3**
b)	It is given that $y = 2\cos 3\left(x - \frac{\pi}{3}\right), 0 \leq x \leq 2\pi$ Find (i) the amplitude.	**1**
	(ii) the period.	**1**
c)	 Two chords CD and AB of a circle are extended to meet at P. If $AB = 7$cm, $BP = 3$cm, $DP = 5$cm find CP.	**1**
d)	The graph of $y = 1 + 2\sin^{-1}(2x-1)$ is shown.  Determine the values of $a, b,$ and c	**2**
e)	Form the cartesian equation by eliminating the parameter, θ, from these parametric equations. $$x = 3\tan\theta \qquad y = 2\sec\theta$$	**2**
f)	Solve $\sin 2\theta + \cos\theta = 0$ for $0 \leq \theta \leq 2\pi$.	**3**
g)	The radius of a spherical balloon is increasing at the rate of 2cm/sec. Find the rate at which the volume of the balloon is increasing when the radius is 10cm (in terms of π).	**2**
	End of Question 11	

Question 12 (15 marks)		Marks
a)	Evaluate $\int_0^{\frac{\pi}{6}} 2\sin^2 x\, dx$	2
b)	By using the substitution $u = \sqrt{x}$, evaluate $$\int_1^4 \frac{dx}{x + \sqrt{x}}$$	3
c)	Two parallel tangents to a circle, centre O, are cut by a third tangent at P and Q.  (i) Copy or trace the diagram into your solution booklet.	
	(ii) Prove $\Delta MOQ \equiv \Delta SOQ$	2
	(iii) Prove that $\angle POQ = 90°$.	2
d)	(i) Express $x = 2\cos 3t - 5\sin 3t$ in the form $x = R\cos(3t + \alpha)$, where t is in radians.	2
	(ii) A particle moves in a straight line and its position at time t is given by $x = 2\cos 3t - 5\sin 3t$. Show that the particle is moving in simple harmonic motion.	1
e)	The coefficients of x^2 and x^{-1} in the expansion of $\left(ax - \frac{b}{x^2}\right)^5$ are the same. Show that $a + 2b = 0$, where a and b are positive integers.	3
	End of Question 12	

Question 13 (15 marks)

a)	In how many ways can a committee of 5 people be formed from a group of 9 people, if 2 people, Harry and Archie, refuse to serve together on the same committee.	**2**
b)	(i) Show by a sketch, without using calculus, that the equation $e^{2x}+4x-5=0$ has only one root.	1
	(ii) Show that this root lies between 0 and 1.	1
	(iii) By taking $x=0.5$ as a first approximation, use one application of Newton's method to find a better approximation of this root to two decimal places.	2
c)	A particle moves in a straight line with velocity v m/sec and acceleration given by $\ddot{x}=2e^{x}$, where x is the displacement from O. The initial velocity is -2 m/sec at the origin.	
	(i) Prove that $v^2=4e^{x}$.	2
	(ii) Hence find the displacement in terms of t.	3
d)	When a body falls, the rate of change of its velocity, v, is given by $\frac{dv}{dt}=-k(v-500)$ where k is a constant.	
	(i) Show that $v=500-500e^{-kt}$ is a possible solution to this equation.	1
	(ii) The velocity after 5 seconds is 21m/sec , find the value of k to 3 significant figures.	1
	(iii) Find the velocity after 20 seconds.	1
	(iv) Explain the effect on the velocity as t becomes large.	1
	End of Question 13	

Question 14 (15 marks) **Marks**

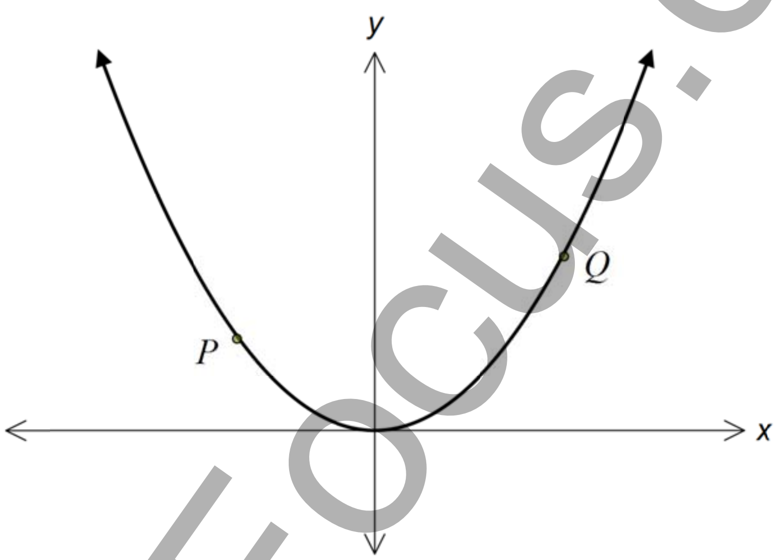
a) A particle is projected with an initial velocity of 60m/sec at an angle of 45° to the horizontal. The equations of motion are:

$$x = 30\sqrt{2}\,t \qquad \text{and} \qquad y = 30\sqrt{2}\,t - 5t^2 \quad \text{(Do not prove these)}$$

(i) How high is the particle after it has travelled 120m horizontally? **2**

(ii) What is the horizontal range of the particle? **1**

b) Two points P and Q move on the parabola $x^2 = 4ay$ so that their x values differ by a.



(i) If P has coordinates (x_1, y_1), show that the midpoint, C, of PQ is **2**

$$\left(\frac{2x_1 + a}{2}, \frac{2x_1^2 + 2ax_1 + a^2}{8a}\right)$$

(ii) Show that the locus of C is another parabola. **2**

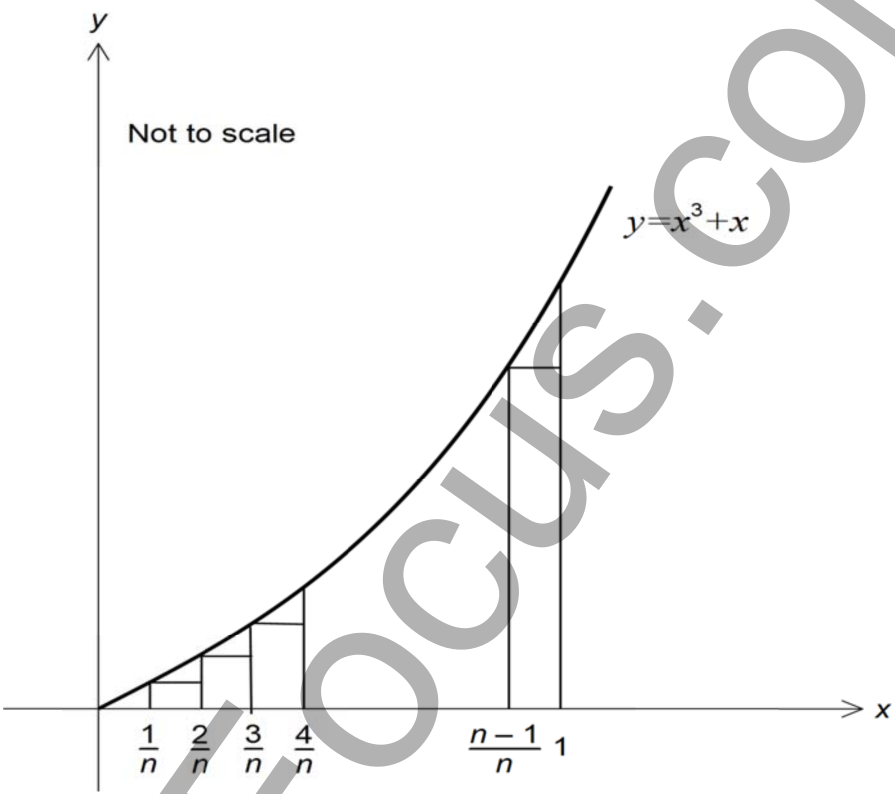
Question 14 continues on page 6

Question 14 (continued)

c) (i) Prove by mathematical induction **3**

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

(ii) The curve $y = x^3 + x$ is drawn and divided into n intervals between 0 and 1. The inner rectangles are completed as shown. **3**



If A_n is the area of these rectangles show that

$$A_n = \frac{1}{n^4}\sum_{r=1}^{n-1} r^3 + \frac{1}{n^2}\sum_{r=1}^{n-1} r$$

(iii) Given that the area bounded by the curve, between $x = 0$ and $x = 1$, and the x axis is $\lim_{n\to\infty} A_n$, find a value for A_n using the result in part (i). **2**

End of Exam

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\qquad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\ a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\ a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\ a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a},\ a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x,\ x > 0$