2012
TRIAL HSC
EXAMINATION

Mathematics Extension 1

SOLUTIONS

Trial HSC Examination - Mathematics Extension 1
Multiple Choice Answer Sheet

Name ________________________________

Completely fill the response oval representing the most correct answer.

1.	A ○	B ○	C ●	D ○
2.	A ○	B ●	C ○	D ○
3.	A ●	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ●
5.	A ●	B ○	C ○	D ○
6.	A ●	B ○	C ○	D ○
7.	A ○	B ●	C ○	D ○
8.	A ○	B ○	C ○	D ●
9.	A ○	B ●	C ○	D ○
10.	A ○	B ○	C ●	D ○

Question 11	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)	$2x+3y=8$ $\qquad$ $x-2y=-5$ $3y=8-2x$ $\qquad$ $2y=x+5$ $y=\frac{8}{3}-\frac{2x}{3}$ $\qquad$ $y=\frac{x}{2}+\frac{5}{2}$ $m_1=-\frac{2}{3}$ $\qquad$ $m_2=\frac{1}{2}$ $\tan\alpha=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $\tan\alpha=\left\|\frac{-\frac{2}{3}-\frac{1}{2}}{1+-\frac{2}{3}\times\frac{1}{2}}\right\|$ $\tan\alpha=\left\|\frac{-\frac{7}{6}}{\frac{2}{3}}\right\|$ $\tan\alpha=\frac{7}{4}$ $\alpha=60°$ So the obtuse angle is $180°-60°=120°$	2	1 for the gradients 1 for correct angle
b)i)	$x=5\sin\theta$...........(1) $y=5\cos\theta+1$................(2) $x=5\sin\theta$........($squaring\ x$) $x^2=25\sin^2\theta$ $x^2=25(1-\cos^2\theta)$ $x^2=25-25\cos^2\theta$ $25\cos^2\theta=25-x^2$..........(3) $y=5\cos\theta+1$.........($squaring$ y) $y^2=25\cos^2\theta+10\cos\theta+1$......(4) From (1) $\cos\theta=\frac{y-1}{5}$...........(5) sub(3) & (5)into(4) $y^2=25-x^2+10\left[\frac{y-1}{5}\right]+1$ $y^2=25-x^2+2y-2+1$ $y^2+x^2-2y-24=0$	2	Alternate solution by making sin θ and cos θ the subject and using $\sin^2\theta+\cos^2\theta=1$ 1 for doing some substitution of *x* into *y* or into $\sin^2\theta+\cos^2\theta=1$ 1 for correctly getting the equation

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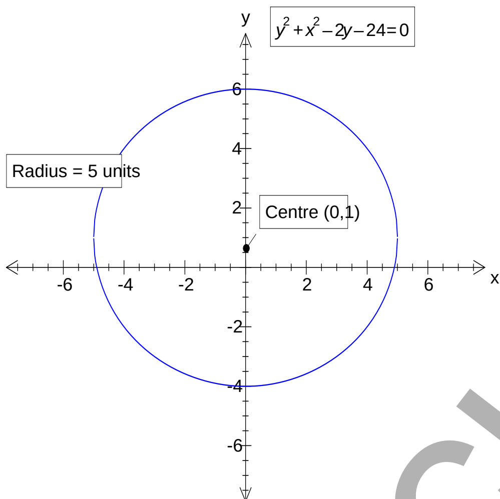
Part	**Solution**	**Marks**	**Comment**
ii)	$y^2+x^2-2y-24=0$ $y^2+x^2-2y+1=24+1$ $x^2+(y-1)^2=25$ *Circle* *Centre*(0,1) *Radius* = 5*units*  	2	1 for realising it is a circle and completing the square 1 for correct graph with centre & radius
c)i)	P(correct) = 0.25 P(incorrect) = 0.75 ${}^{10}_{10}C(0.25)^{10}(0.75)^1$ $=\dfrac{1}{1048576}$	1	
ii)	${}^{10}_{2}C(0.25)^2(0.75)^8$ $=0.282$	1	
iii)	${}^{10}_{8}C(0.25)^8(0.75)^2+{}^{10}_{9}C(0.25)^9(0.75)^1+{}^{10}_{10}C(0.25)^{10}(0.75)^0$ $=4.15\times10^{-4}$	2	

Question 11	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
d)	$\int 3x\sqrt{4-x}\,dx$ $u=4-x \qquad x=4-u$ $du=-1dx$ $\int 3x\sqrt{4-x}\,dx$ $=3\int x\sqrt{4-x}\,dx$ $=-3\int (4-u)\sqrt{u}\,du$ $=-3\int (4-u)u^{\frac{1}{2}}\,du$ $=-3\int 4u^{\frac{1}{2}}-u^{\frac{3}{2}}\,du$ $=-3\left[\frac{8}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right]+C$ $=-8\sqrt{(4-x)^3}+\frac{6\sqrt{(4-x)^5}}{5}+C$	3	1 1 1
e)	$\left(3+\frac{1}{x}\right)^2+4\left(3+\frac{1}{x}\right)-21=0$ $Let\ y=\left(3+\frac{1}{x}\right)$ $y^2+4y-21=0$ $(y+7)(y-3)=0$ $y=-7\ or\ 3$ $\therefore 3+\frac{1}{x}=-7$ or $3+\frac{1}{x}=3$ $\frac{1}{x}=-10$ $\frac{1}{x}=0$ $x=-\frac{1}{10}$ No Solution So $x=-\frac{1}{10}$ is the only solution.	2	1 for using a substitution and finding values 1 for correct solution after resubstitution
		/15	

Question 12	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)	$4^n > 1+3n$ for $n>1$ *ie* $4^n - 3n - 1 > 0$ Step 1 Show true for $n>1$ *ie* $n=2$ $4^2 - 3\times 2 - 1 > 0$ $9 > 0 \therefore$ *true* Step 2 Assume true for $n=k$ $4^k - 3k - 1 > 0$ Step 3 Prove true for $n=k+1$ $4^{k+1} - 3(k+1) - 1 > 0$ $4\times 4^k - 3k - 3 - 1 > 0$ $4\times 4^k - 3k - 4 > 0$ $4(4^k - 3k - 1) + 9k > 0$ since $4^k - 3k - 1 > 0$ and $9k > 0$ Hence if the statement is true for $n=k$, then it is also true for $n=k+1$ The statement is true for $n=2$ and so it is true for $n=3$ and so on. Hence true for all $n>1$	3	1 mark for steps 1 and 2 2 marks for step 3, including conclusion.
b)i)	P(success) = (0.65) P(notsuccess) = (0.35) ${}^{10}_{9}C(0.65)^9(0.35)^1$ $= 0.072$ $= 7\%$	1	

Question 12	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	We need the greatest term in the expansion $\frac{n-r+1}{r}\times\frac{b}{a}\geq 1$ $\frac{10-r+1}{r}\times\frac{0.35}{0.65}\geq 1$ $r\leq 3.85$ $\therefore r=3$ ${}^{10}_{3}C(0.65)^7(0.35)^3$ $\therefore$ 7 will be the most likely number of people to sign up.	2	1 for finding the greatest term 1 for correct number of sign ups.
c)	$\int_0^{\frac{\pi}{4}}\sin x\cos^2 x\ \ dx$ $=\left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{4}}$ $=-\frac{1}{3}\left[\left(\cos\frac{\pi}{4}\right)^3-(\cos 0)^3\right]$ $=-\frac{1}{3}\left[\left(\frac{1}{\sqrt{2}}\right)^3-1\right]$ $=-\frac{1}{3}\left[\frac{1}{2\sqrt{2}}-1\right]$ $=-\frac{1}{6\sqrt{2}}+\frac{1}{3}$ $=-\frac{-1+2\sqrt{2}}{6\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $=\frac{4-\sqrt{2}}{12}$	2	Can also be done by substitution 1 correct integration 1 for answer in any form

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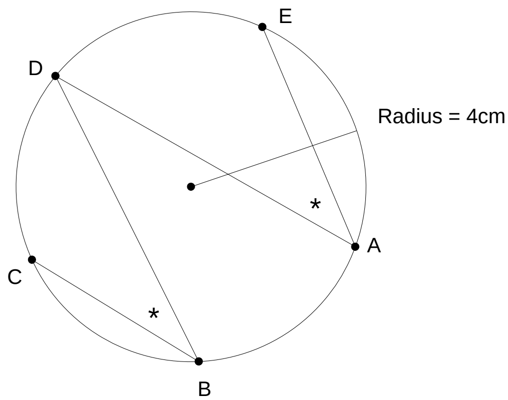
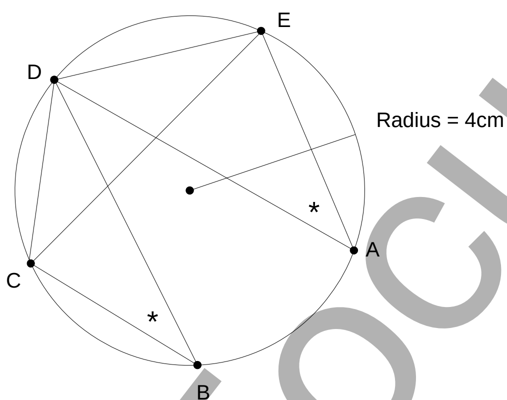
Part	**Solution**	**Marks**	**Comment**
d)i)	 	1	Must show all information, including radius
ii)	 $\angle DBC = \angle DAE (given)$ $ArcCD = ArcDE$ (converse of angles on the same arc) $\therefore CD = DE$ (equal arcs subtend equal chords) $\therefore \Delta CDE$ is isosceles as $CD = DE$	2	1 1

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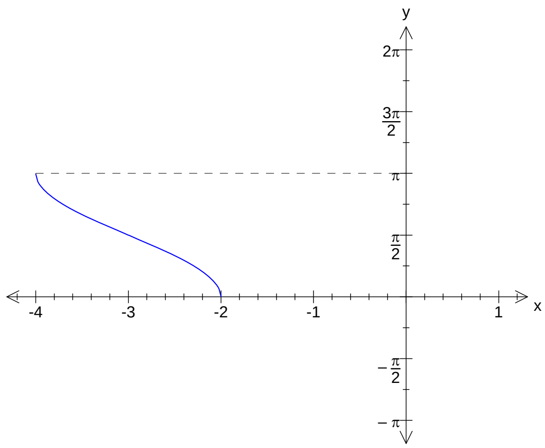
Part	**Solution**	**Marks**	**Comment**
e)	 	2	
f)	$\left(3x-\frac{4}{5x^2}\right)^9$ $T_{k+1}={}^nC_k a^{n-k}x^k$ $T_6={}^9C_5(3x)^{9-5}(-\frac{4}{5x^2})^5$ $=126(81x^4)-\frac{1024}{3125x^{10}}$ $=-\frac{10450944}{3125x^6}$	2	1 1
		/15	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)	$\int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ $\sqrt{4-2x^2}$ $=\sqrt{2(2-x^2)}$ $=\sqrt{2}\sqrt{2-x^2}$ $\therefore \int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ $=\frac{1}{\sqrt{2}}\int_0^1 \frac{1}{\sqrt{2-x^2}}$ $=\frac{1}{\sqrt{2}}\left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^1$ $=\frac{1}{\sqrt{2}}\left(\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-\sin^{-1}\left(\frac{0}{\sqrt{2}}\right)\right]\right)$ $=\frac{1}{\sqrt{2}}\left[\frac{\pi}{4}-0\right]$ $=\frac{\pi}{4\sqrt{2}}$ $=\frac{\sqrt{2}\pi}{8}$	2	1 for getting in correct form and using standard integrals 1 answer
b)i)	$T=R+Ce^{-kt}$ $\frac{dT}{dt}=kCe^{-kt}$ and $T-R=Ce^{-kt}$ $\therefore \frac{dT}{dt}=-k(T-R)$ $\therefore T=R+Ce^{-kt}$ is a solution to the differential equation	1	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	$When\ \ t=0\ \ T=540°\ \ R=20°$ $540=20+Ce^{0}$ $520=Ce^{0}$ $C=520$ $\therefore T=20+520e^{-kt}$ $When\ \ t=50\ \ T=100°\ \ R=20°$ $100=20+520e^{-50k}$ $\frac{80}{520}=-50k$ $k=\frac{\ln\left[\frac{8}{52}\right]}{-50}$ $k=0.037436043$ $\therefore T=20+520e^{-0.037436043t}$ $When\ \ T=40°\ \ R=20°$ $40=20+520e^{-0.037436043\times t}$ $t=\frac{\ln\left[\frac{1}{26}\right]}{0.037436043}$ $t=87\min utes$ Extra time $87-50=37$ So Storm must wait another 37 minutes for the Toffee to cool enough	3	1 for C 1 for k 1 for correct answer must have used full value of k
iii)	$40=25+520e^{-0.037436043\times t}$ $t=\frac{\ln\left[\frac{3}{104}\right]}{0.037436043}$ $t=95\text{minutes}$ $\therefore$ it would take longer for the toffee to cool in a room at 25°	2	1 for the calculation or other explanation 1 for a statement stating the result

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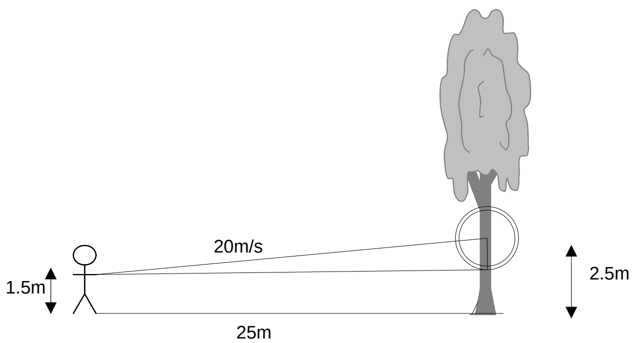
Part	**Solution**	**Marks**	**Comment**
c)	$y=\ln x-1$ $y+1=\ln x$ $e^{y+1}=x$ $x^2=\left(e^{y+1}\right)^2$ $=e^{2y+2}$ $V=\pi\int_a^b x^2 dy$ $=\pi\int_0^1 e^{2y+2}dy$ $=\pi\left(\frac{e^{2y+2}}{2}\right)_0^1$ $=\pi\left\{\left[\frac{e^4}{2}\right]-\left[\frac{e^2}{2}\right]\right\}$ $=\frac{\pi}{2}\left(e^4-e^2\right)units^3$	2	1 for finding x^2 1 for answer
d)i)	$P(x)=x^4-2x^3+5x^2-16x+12$ $P(1)=1^4-2\times1^3+5\times1^2-16\times1+12$ $P(1)=1-2+5-16+12$ $P(1)=0\ \therefore (x-1)$ is a factor $P(2)=2^4-2\times2^3+5\times2^2-16\times2+12$ $P(2)=16-16+20-32+12$ $P(2)=0\ \therefore (x-2)$ is a factor $\therefore (x-1)(x-2)$ is a factor of $P(x)=x^4-2x^3+5x^2-16x+12$	1	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	$(x-1)(x-2)=x^2-3x+2$ $$\begin{array}{r} x^2+x+6 \\ x^2-3x+2\overline{\big)\,x^4-2x^3+5x^2-16x+12} \\ x^4-3x^3+2x^2 \qquad\qquad \\ \hline x^3+3x^2-16x \qquad \\ x^3-3x^2+2x \qquad \\ \hline 6x^2-18x+12 \\ 6x^2-18x+12 \\ \hline 0 \end{array}$$ $\therefore$ The remaining factor is (x^2+x+6) which cannot be reduced to linear factors.	2	1 1
e)i)	$A=\frac{1}{2}ab\sin C$ $A=\frac{1}{2}\times(x-2)\times(x-2)\times\sin 60°$ $A=\frac{1}{2}(x-2)^2\times\frac{\sqrt{3}}{2}$ $A=\frac{\sqrt{3}(x-2)^2}{4}$	1	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	$Let\ s = x - 2\ \ (side\ length)$ $\frac{ds}{dx} = 1$ and $\frac{ds}{dt} = 5mm/s = 0.5cm/s$ $\frac{ds}{dt} = \frac{ds}{dx} \times \frac{dx}{dt}$ $0.5 = 1 \times \frac{dx}{dt}$ $\frac{dx}{dt} = 0.5$ $A = \frac{\sqrt{3}(x-2)^2}{4}$ $\frac{dA}{dx} = \frac{\sqrt{3}(x-2)}{2}$ $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $\frac{dA}{dt} = \frac{\sqrt{3}(x-2)}{2} \times 0.5$ $when$ s=10, $x = 12$ $\frac{dA}{dt} = \frac{\sqrt{3}(12-2)}{2} \times 0.5$ $= \frac{5\sqrt{3}}{2} cm^2/s$ So the area of the triangle is increasing at $4.33cm^2/s$	2	1 Both exact or decimal answer OK 1
		/15	

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)	 	2	1 for each
i)	Vertical Motion $\ddot{y} = -g = -9.8ms^{-2}$ $\therefore \dot{x} = \int -9.8dt$ $= -9.8t + C_2$ $When \quad t = 0 \quad \dot{y} = V\sin\theta$ $So \quad C = V\sin\theta$ $\dot{y} = V\sin\theta$ $20\sin\alpha = -9.8t + C_2$ $C_2 = 20\sin\alpha$ $\dot{y} = -9.8t + 20\sin\alpha$ $y = \int -9.8t + 20\sin\alpha dt$ $= -4.9t^2 + 20t\sin\alpha + C_3$ $When \quad t = 0 \quad y = 1.5$ $0 = 0 + C_1$ $\therefore C_3 = 1.5$ $\therefore y = -4.9t^2 + 20t\sin\alpha + 1.5$		1 for using correct integrations 1 for final result.

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	$x = 20t\cos\alpha$.......(1) $y = -4.9t^2 + 20t\sin\alpha + 1.5$.......(2) $From(1)$ $t = \dfrac{x}{20\cos\alpha}$.......(3) sub (3) into (2) $y = -4.9\left[\dfrac{x}{20\cos\alpha}\right]^2 + 20\left[\dfrac{x}{20\cos\alpha}\right]\sin\alpha + 1.5$ $= \dfrac{-4.9x^2}{400\cos^2\alpha} + \dfrac{x\sin\alpha}{\cos\alpha} + 1.5$ $= \dfrac{-4.9}{400}x^2\sec^2\alpha + x\tan\alpha + 1.5$ $= \dfrac{-4.9}{400}x^2(1+\tan^2\alpha) + x\tan\alpha + 1.5$ For the ball to hit the target $x = 25$ *and* $y = 2.5$ $2.5 = \dfrac{-4.9}{400}25^2(1+\tan^2\alpha) + 25\tan\alpha + 1.5$ $2.5 = -7.65625(1+\tan^2\alpha) + 25\tan\alpha + 1.5$ $0 = -7.65625\tan^2\alpha - 25\tan\alpha + 8.65625$	3	1
ii)	Let $x = \tan\alpha$ Solve using quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{25 \pm \sqrt{25^2 - (4\times 7.65625 \times 8.65625}}{2\times 7.65625}$ $x = \dfrac{25 \pm 18.97}{15.3125}$ $x = 2.87$ or 0.39 $\tan\alpha = 2.87$ *or* $\tan\alpha = 0.39$ $\alpha = 70°47'$ *or* $21°29'$ However to avoid the air resistance Zanthie must shoot at an angle of 21°		1 1

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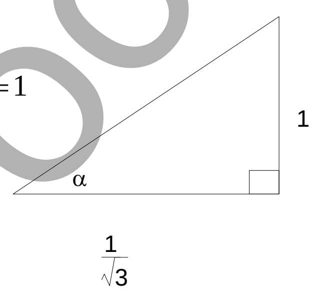
Part	**Solution**	**Marks**	**Comment**
b)	$(1+x)^{n+4} = (1+x)^n(1+x)^4$ $\binom{n+4}{r} = \binom{n}{r} + 4\binom{n}{r-1} + 6\binom{n}{r-2} + 4\binom{n}{r-3} + \binom{n}{r-4}$ *LHS* $(1+x)^{n+4} = 1 + \binom{n+4}{r}x + ... + \binom{n+4}{r}x^r + ...$ coefficient of $x^r = \binom{n+4}{r}$ *RHS* $(1+x)^n(1+x)^4 = \left[1 + \binom{n}{1}x + ... + \binom{n}{r}x^r + ...\right] \times$ $\left[1 + \binom{4}{1}x + \binom{4}{2}x^2 + \binom{4}{3}x^3 + \binom{4}{4}x^4\right]$ coefficient of $x^r = \binom{n}{r} + \binom{n}{r-1}\binom{4}{1} + \binom{n}{r-2}\binom{4}{2} +$ $\binom{n}{r-3}\binom{4}{3} + \binom{n}{r-4}\binom{4}{4}$ $= \binom{n}{r} + 4\binom{n}{r-1} + 6\binom{n}{r-2} + 4\binom{n}{r-3} + \binom{n}{r-4}$ Because coefficients of x^r in both expansions are the same it follows $\binom{n+4}{r} = \binom{n}{r} + 4\binom{n}{r-1} + 6\binom{n}{r-2} + 4\binom{n}{r-3} + \binom{n}{r-4}$	2	1

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
c)	Equation of the chord of contact $xx_1 = 2a(y + y_1)$ This passes through $B(0\ ,\ 2a)$ $0x_1 = 2a(2a + y_1)$ $0 = 4a^2 + 2ay_1$ $\frac{2ay_1}{2a} = \frac{-4a^2}{2a}$ $y_1 = -2a$ Locus of the Midpoint AB $A(x_1\ ,\ -2a)$ & $B(0\ ,\ 2a)$ $y = \frac{-2a + 2a}{2}$ $x = \frac{x_1}{2}$ $y = 0$ $\therefore y = 0$ is the equation of the locus of the midoint AB	2	1 for finding y_1 1 for the equation of the locus
d)i)	$R\sin(4t - \alpha) = R\sin 4t\cos\alpha - R\cos 4t\sin\alpha$ *If* $R\sin(4t - \alpha) = \frac{1}{\sqrt{3}}\sin 4t - \cos 4t$ *then* $r\cos\alpha = \frac{1}{\sqrt{3}}$ and $r\sin\alpha = 1$  continued over	2	

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Part	**Solution**	**Marks**	**Comment**
	$R=\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+1}$ $=\sqrt{\frac{1}{3}+1}$ $=\sqrt{\frac{4}{3}}$ $=\frac{2}{\sqrt{3}}$ $\tan\alpha=\frac{R\sin\alpha}{R\cos\alpha}$ $=\frac{1}{\frac{1}{\sqrt{3}}}$ $=\sqrt{3}$ $\therefore \tan\alpha=\sqrt{3}$ $\alpha=\frac{\pi}{3}$ $\therefore \frac{\sin 4t}{\sqrt{3}}-\cos 4t=R\sin(4t-\alpha)$ $\frac{\sin 4t}{\sqrt{3}}-\cos 4t=\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})$		1 for deriving R 1 for deriving α
ii)	$x=4+\frac{\sin 4t}{\sqrt{3}}-\cos 4t$ *from part(i)* $x=4+\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})$ $\dot{x}=\frac{8}{\sqrt{3}}\cos(4t-\frac{\pi}{3})$ $\ddot{x}=-\frac{32}{\sqrt{3}}\sin(4t-\frac{\pi}{3})$ $\ddot{x}=-16\left[\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})\right]$ and from (1) $x-4=\frac{2}{\sqrt{3}}\sin(4t-\frac{\pi}{3})$ $\ddot{x}=-16(x-4)$	2	1 1

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Part	**Solution**	**Marks**	**Comment**
iii)	The maximum speed occurs when the particle is at the centre of motion. When $x = 4$ $4 = 4 + \frac{2}{\sqrt{3}}\sin(4t - \frac{\pi}{3})$ $\frac{2}{\sqrt{3}}\sin(4t - \frac{\pi}{3}) = 0$ $\sin(4t - \frac{\pi}{3}) = 0$ $4t - \frac{\pi}{3} = 0, \pi, 2\pi, ...$ $4t = \frac{\pi}{3}, \frac{4\pi}{3}, ...$ $t = \frac{\pi}{12}, \frac{\pi}{3}, ...$ The particle first reaches maximum speed when $t = \frac{\pi}{12}$ seconds	2	1 1
		/15	