2011
TRIAL HSC
EXAMINATION

Mathematics
Extension 1

SOLUTIONS

Question 1	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
a)	Domain and range of $y = 4\cos^{-1}\left(\frac{3x}{2}\right)$ $y = \cos^{-1}(x)$ has domain $-1 \le x \le 1$ and range $0 \le y \le \pi$ So for $y = 4\cos^{-1}\left(\frac{3x}{2}\right)$ i.e. $\frac{y}{4} = \cos^{-1}\left(\frac{3x}{2}\right)$ Domain is given by $-1 \le \frac{3x}{2} \le 1$ $-2 \le 3x \le 2$ $-\frac{2}{3} \le x \le \frac{2}{3}$ Range given by $0 \le \frac{y}{4} \le \pi$ $0 \le y \le 4\pi$	2	1 for range and 1 for domain Or if both answers wrong 1 mark for reasonable attempt to deduce D and R.
b)	$x - 3y - 2 = 0$ $3y = x - 2$ $y = \frac{x}{3} - \frac{2}{3}$ Gradient $= \frac{1}{3}$ $x - 2y = 0$ $2y = x$ $y = \frac{x}{2}$ Gradient $= \frac{1}{2}$. $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$ $= \left\lvert \frac{\frac{1}{2} - \frac{1}{3}}{1 + \frac{1}{2}.\frac{1}{3}} \right\rvert$ $= \left\lvert \frac{\frac{1}{6}}{\frac{7}{6}} \right\rvert$ $= \frac{1}{7}$ $\theta = \tan^{-1}\left(\frac{1}{7}\right)$ $= 8^\circ$ (nearest degree)	2	1 for gradients of the two lines 1 for evaluating the angle

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c)	Root of $y = e^x - 2x^2$ using $x = 2.5$ $f(x) = e^x - 2x^2$ $f'(x) = e^x - 4x$ $f(2.5) = -0.318$ $f'(2.5) = 2.182$ $x_1 = 2.5 - \dfrac{f(2.5)}{f'(2.5)}$ $= 2.5 - \dfrac{-0.318}{2.182}$ $= 2.5 - (-0.145)$ $= 2.645$	2	1 for values of $f(2.5)$ and $f'(2.5)$ 1 use of N M to obtain answer.
d)	To find asymptote to $y = \dfrac{3x^2 - 2x + 1}{x^2 - x}$ For horizontal asymptote divide $x^2 - x \overline{)\,3x^2 - 2x + 1}$ quotient 3 $3x^2 - 3x$ $x + 1$ So $y = 3 + \dfrac{x+1}{x^2 - x}$ as $x \to \pm\infty$; $\dfrac{x+1}{x^2 - x} \to 0$ so $y \to 3$ Horizontal asymptote is $y = 3$ Discontinuities when $x^2 - x = 0$ so vertical asymptotes where $x(x-1) = 0$ i.e. when $x = 0$ and $x = 1$ Vertical asymptotes where $x = 0$ and $x = 1$.	3	1 mark for each asymptote. Or 2 marks for horizontal asymptote correct and attempt at discontinuities Or 1 mark if some progress made toward both horizontal and vertical but answers wrong.

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Part	**Solution**	**Marks**	**Comment**
e)	$\frac{2x}{x-1} \geq 6 \quad ; x \neq 1$ $\frac{2x}{x-1}(x-1)^2 \geq 6(x-1)^2$ $2x(x-1) \geq 6(x^2-2x+1)$ $2x^2-2x \geq 6x^2-12x+6$ $0 \geq 4x^2-10x+6$ $4x^2-10x+6 \leq 0$ $4x^2-4x-6x+6 \leq 0$ $4x(x-1)-6(x-1) \leq 0$ $2(2x-3)(x-1) \leq 0$ $1 \leq x \leq \frac{3}{2}$ But $x \neq 1$ so $1 < x \leq \frac{3}{2}$	3	1 for multiplying by $(x-1)^2$ correctly 1 for solving inequality 1 for final answer including $x \neq 1$.
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Question 2	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
a)	$y = \frac{x^2}{8}$ $\frac{dy}{dx} = \frac{2x}{8} = \frac{x}{4}$ when $x = 4p$ $\frac{dy}{dx} = \frac{4p}{4} = p$ Gradient of normal = $-\frac{1}{p}$ Equation $y - 2p^2 = -\frac{1}{p}(x - 4p)$ $py - 2p^3 = -x + 4p$ $x + py = 2p^3 + 4p$	3	1 for gradient 1 for sub in equation of line. 1 for simplifying
b)	$\cos 3x = \cos(2x + x)$ $= \cos 2x.\ \cos x - \sin 2x.\ \sin x$ $= (2\cos^2 x - 1).\cos x - 2\sin x.\cos x.\sin x$ $= 2\cos^3 x - \cos x - 2\cos x.\sin^2 x$ $= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x)$ $= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$ $= 4\cos^3 x - 3\cos x$	3	1 for use of the sum result to start the expression. 2 for using the double angle and eliminating sinx 3 for simplifying and obtaining final result.

Question 2	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
c)	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{(x+h)^3 + (x+h) - (x^3 + x)}{h}$ $= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + x + h - x^3 - x}{h}$ $= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 + h}{h}$ $= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2 + 1)}{h}$ $= \lim_{h \to 0} (3x^2 + 3xh + h^2 + 1)$ $= 3x^2 + 1$	3	1 if substitute correctly into formula 1 for expanding 1 for applying the limit correctly
d)	$T_{k+1} = {}^{15}\mathbf{C}_k \left(\frac{4}{x}\right)^{15-k} (x^2)^k$ $= {}^{15}\mathbf{C}_k 4^{15-k} x^{k-15} . x^{2k}$ $= {}^{15}\mathbf{C}_k 4^{15-k} . x^{3k-15}$ For term independent of x $3k - 15 = 0$ $3k = 15$ $k = 5$ $T_6 = {}^{15}\mathbf{C}_5 \left(\frac{4}{x}\right)^{15-5} (x^2)^5$ $= {}^{15}\mathbf{C}_5 \frac{4^{10}}{x^{10}} . x^{10}$ $= {}^{15}\mathbf{C}_5 (4^{10})$ = 3003(1 048 576) = 3 148 873 728	3	1 mark for general term 1 mark for setting power = 0 and solve. 1 mark for term (left in ${}^{n}\mathbf{C}_r$ form is okay)
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Question 3	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
a)	$\frac{d}{dx}(cos^{-1}(4x)) = \frac{-1}{\sqrt{\left(\frac{1}{4}\right)^2 - x^2}}$ $= \frac{-1}{\sqrt{\frac{1}{16} - x^2}}$ $= \frac{-1}{\sqrt{\frac{1-16x^2}{16}}}$ $= \frac{-4}{\sqrt{1-16x^2}}$	2	1 for initial substitution in rule. 1 for expanding and simplifying
b)	A(-4, 3) and B(-8, -9) Externally so use 2 : -3 $x = \frac{-3(-4) + 2(-8)}{2 + (-3)}$ $y = \frac{-3(3) + 2(-9)}{2 + (-3)}$ $= \frac{12 - 16}{-1}$ $= \frac{-9 - 18}{-1}$ $= \frac{-4}{-1}$ $= \frac{-27}{-1}$ $= 4$ $= 27$ P is the point (4, 27)	2	1 mark for x 1 mark for y

Question 3	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
c)	$u = 1 + 3x^2$ so $\frac{du}{dx} = 6x$ and $du = 6x\,dx$ When $x = 1, u = 4$ and when $x = 4, u = 49$ $\int_1^4 2x\sqrt{1+3x^2}\,dx = \frac{1}{3}\int_1^4 6x\sqrt{1+3x^2}\,dx$ $= \frac{1}{3}\int_4^{49}\sqrt{u}\,du$ $= \frac{1}{3}\int_4^{49} u^{\frac{1}{2}}\,du$ $= \frac{1}{3}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_4^{49}$ $= \frac{1}{3}\left(\frac{2}{3}\sqrt{49^3} - \frac{2}{3}\sqrt{4^3}\right)$ $= \frac{2}{9}(343 - 8)$ $= 74\frac{4}{9}$	4	1 for correct value of du 1 for limits 1 for substitution 1 for integral

Question 3	**Trial HSC Examination - Mathematics Extension 1**		**2011**
Part	**Solution**	**Marks**	**Comment**
d)	Step 1 show true of n = 1 $4^1 + 8 = 12$ 12 is divisible by 6 $\therefore$ true for $n = 1$ Step 2 Assume true for $n = k$ and prove true for $n = k + 1$ Assume that $4^k + 8 = 6p$ (p is a positive integer) Want to prove that $4^{k+1} = 6q$ (q is a positive integer) $LHS = 4^{k+1} + 8$ $= 4(4^k) + 8$ $= 4(4^k + 8 - 8) + 8$ $= 4(4^k + 8) - 32 + 8$ $= 4(6p) - 24$ $= 24p - 24$ $= 6(4p - 4)$ $= 6q$ (q is positive integer > 1 since $p > 1$) $= RHS$ Step 3 Using the principle of induction Since true for $n = 1$, and since if true for $n = k$ is also true for $n = k + 1$, by induction is true for all $n \geq 1$.	4	1 mark for step 1 2 marks for step 2 Or 1 mark for significant progress or a simple error in step 2 1 mark for step 3
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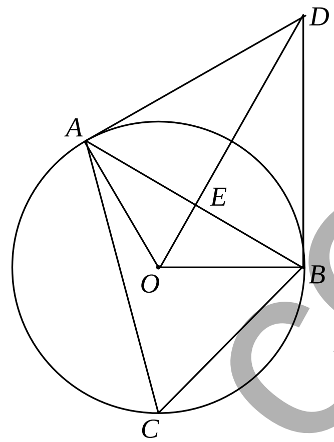
Question 4	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
a) (i)	 Aim prove $\angle AOB = 2 \times \angle DAB$ $\angle DAB = \angle ACB$ ($\angle$bet tangent & chord $=\angle$ in alt segment) $\angle AOB = 2 \times \angle ACB$ ($\angle$ at centre is twice $\angle$ at circ on same arc) $\therefore \angle AOB = 2 \times \angle DAB$ (since $\angle DAB = \angle ACB$)	2	2 marks for complete proof. 1 mark if some relevant facts are stated or proof is incomplete
(ii)	$\angle DAO = \angle DBO = 90^\circ$(tangent perpendicular to radius $\therefore \angle DAO + \angle DBO = 180^\circ$ (sum of two right angles) $\therefore$ opposite angles of *AOBD* are supplementary $\therefore$ *AOBD* is a cyclic quadrilateral.	1	1 mark as long as statement that opposite angles are supplement ary, and why.
(iii)	Aim: Prove that *E* is the midpoint of *AB*. $AO = BO$ (equal radii) $AD = BD$ (tangents from an external point are equal in length.) $\therefore$ *AOBD* is a kite $\therefore$ *OD* bisects AB (symmetry of a kite) $\therefore$ E is midpoint of AB.	2	Can also be done by isosceles triangles 2 marks for complete proof. 1 mark if some relevant facts are stated or proof is incomplete

Question 4	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
b) (i)	$x = 30t \cos 72^\circ$ $y = 30t \sin 72^\circ - 4.9t^2$ At ground level when $y = 0$ $y = 30t \sin 72^\circ - 4.9t^2 = 0$ $t(30\sin 72^\circ - 4.9t) = 0$ $t = 0$ and $30\sin 72^\circ - 4.9t = 0$ $4.9t = 30\sin 72^\circ$ $t = \dfrac{30\sin 72^\circ}{4.9}$ $t = 5.8$ sec The ball is in the air for 5.8 sec.	2	2 marks for full solution 1 mark is simple mistake made in correct method
(ii)	Maximum height occurs when vertical motion = 0 $y = 30t \sin 72^\circ - 4.9t^2$ $y' = 30\sin 72^\circ - 9.8t$ $y' = 0$ $30\sin 72^\circ - 9.8t = 0$ $9.8t = 30\sin 72^\circ$ $t = \dfrac{30\sin 72^\circ}{9.8}$ $t = 2.9$ sec Height : $y = 30(2.9) \sin 72^\circ - 4.9(2.9)^2$ $= 41.5\ m$ Maximum height is 41.5 metres.	2	Can also obtain t = 2.9 by halving answer from part (i) 2 marks for full solution 1 mark is simple mistake made in correct method

Question 4	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
(iii)	Dist from O to Q is x value For 72^o $x = 30(5.8)\ cos\ 72^\circ$ $= 53.8$ metres If angle changes to 60^o $y = 30t\ sin\ 60^\circ - 4.9t^2$ At ground level when $y = 0$ $y = 30t\ sin\ 60^\circ - 4.9t^2 = 0$ $t(30sin\ 60^\circ - 4.9t) = 0$ $30sin\ 60^\circ - 4.9t = 0$ $4.9t = 30sin\ 60^\circ$ $t = \frac{30sin\ 60^\circ}{4.9}$ $t = 5.3$ sec $x = 30(5.3)\ cos\ 60^\circ$ $x = 79.5$ metres The horizontal distance would be an extra 25.7 metres.	3	3 marks for full solution 2 marks for correct solution with a minor error. 1 mark for attempted solution with multiple errors 1 mark if just sub 60^o into original equation for x
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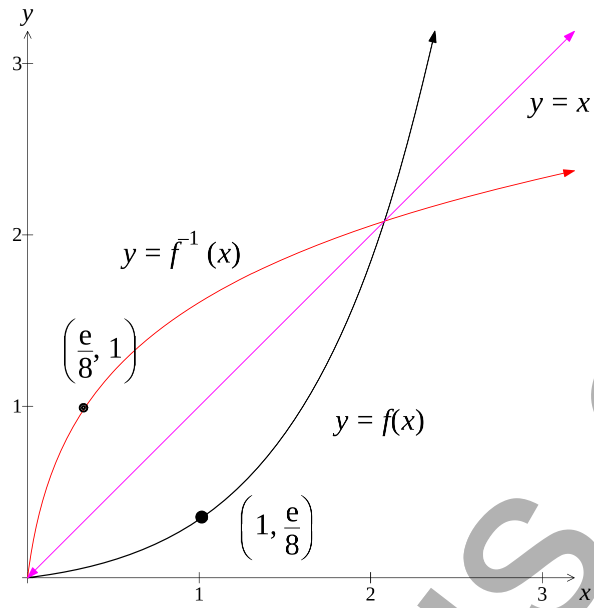
Question 5	**Trial HSC Examination - Mathematics Extension 1**		**2011**
Part	**Solution**	**Marks**	**Comment**
a)	$p(x) = x^3 + ax + b$ has $(x-5)$ as one of its factors $\therefore \quad p(5) = 0$ $125 + 5a + b = 0$ Remainder = -60 when divided by $(x+5)$. $\therefore \quad p(-5) = -60$ $-125 - 5a + b = -60$ Solve simultaneously to find *a* and *b*. $5a + b + 125 = 0 \quad ①$ $-5a + b - 65 = 0 \quad ②$ $2b + 60 = 0 \quad ① + ②$ $2b = -60$ $b = -30$ $5a - 30 + 125 = 0$ $5a = -95$ $a = -19$	3	1 mark for use of factor theorem 1 mark for use of remainder theorem 1 mark for solving simultaneously Part marks as appropriate if other approaches taken
b) (i)	P(win first and last) = P(WLLLW) $= 0.3 \times 0.7 \times 0.7 \times 0.7 \times 0.3$ $= 0.03087$	1	Give mark if calculation only is given.
(ii)	$P(W=2) = {}^5\mathbf{C}_2\, 0.3^2 \times 0.7^3$ $= 10 \times 0.03087$ $= 0.3087$	1	Give mark if calculation only is given.
(iii)	Wins no more than 2 games = P(W=0) +P(W=1) +P(W=2) $= {}^5\mathbf{C}_0\, 0.3^0 \times 0.7^5 + {}^5\mathbf{C}_1\, 0.3^1 \times 0.7^4 + {}^5\mathbf{C}_2\, 0.3^2 \times 0.7^3$ = 0.16807 + 0.36015 + 0.3087 =0.83692	2	2 marks for full solution 1 mark if missed a term or used wrong terms, or arithmetic error.

Question 5	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
c)	$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} cos^2 2x\, dx = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 1 + cos\, 4x\, dx$ $= \frac{1}{2}\left[x + \frac{1}{4} sin\, 4x \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$ $= \left(\frac{\pi}{4} + \frac{1}{8} sin\, 2\pi\right) - \left(\frac{\pi}{6} + \frac{1}{8} sin\, \frac{4\pi}{3}\right)$ $= \frac{\pi}{4} + \frac{1}{8} \times 0 - \frac{\pi}{6} - \frac{1}{8} \times \frac{\sqrt{3}}{2}$ $= \frac{\pi}{12} - \frac{\sqrt{3}}{16}$ (= 0.154)	2	1 mark for obtaining integral 1 mark for correct substitution
d) (i)	$y = \frac{xe^x}{2}$ $y' = \frac{x}{2} \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}\left(\frac{x}{2}\right)$ $= \frac{x}{2} \cdot e^x + e^x \cdot \frac{1}{2}$ $= \frac{xe^x + e^x}{2}$ Now for $x > 0$, $e^x > 1$ $\therefore$ $x\, e^x > 0$ (positive × positive) $\therefore$ $xe^x + e^x > 0$ (positive + positive) $\therefore$ $\frac{xe^x + e^x}{2} > 0$ (positive ÷ positive)	1	1 if differentiation and explanation both correct.
(ii)	A function only has an inverse if it is a one to one function for the domain given. i.e. a horizontal line only intersects it at most in one point. Since $y = f(x)$ is defined for x > 0 it is always increasing, so has no turning points and a horizontal line will intersect it at most once.	1	1 mark for mention of 1-1 function increasing function horizontal line test

Question 5	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
(iii)		1	As long as graph appears to be reflection of original curve in $y = x$ give the mark
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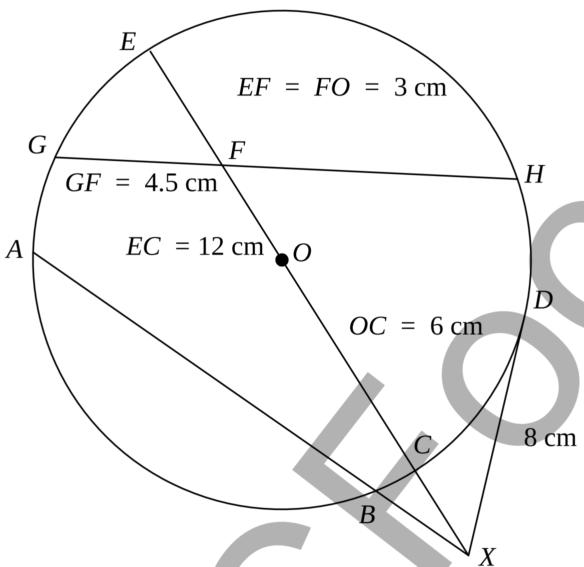
Question 6	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
a) (i)	Ways of arranging 8 different tiles = 8! = 40 320	1	
(ii)	Ways of arranging 4 chosen from 8 = $^{8}\mathbf{P}_4$ = 1680	1	
(iii)	Ways of choosing 4 from 8 = $^{8}\mathbf{C}_4$ = 70	1	
(iv)	Ways of arranging 8 different tiles with 2 M's and 3 I's = $\frac{8!}{3! \times 2!} = 3\ 360$	1	
(v)	With M's placed at the ends, leaves 6 to arrange, with 3 I's. Ways of arranging 6 different tiles with 3 I's $= \frac{6!}{3!} = 120$	1	
b)	 		
(b) (i)	$DX^2 = CX \,.\, XE$ (ratio of secant = square of tangent) $8^2 = CX.\,(CX + 12)$ $64 = CX^2 + 12CX$ $CX^2 + 12CX - 64 = 0$ $(CX + 16)(CX - 4) = 0$ $CX = -16$ or $CX = 4$ Take positive value $CX = 4$	2	1 for use of correct theorem 1 for solving for CX

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Part	**Solution**	**Marks**	**Comment**
(b) (ii)	$GF.\ FH = EF.\ FC$ (Products of intercepts on chords are equal) $4.5.\ FH = 3\ .\ 9$ $4.5\ FH = 27$ $FH = 6$ $GH = FH + GF$ $= 6 + 4.5$ $GH = 10.5$ cm	2	1 for use of correct theorem. 1 for solving for GH
c) i)	$x = 2\cos\left(3t + \frac{\pi}{3}\right)$ $\dot{x} = -2\sin\left(3t + \frac{\pi}{3}\right).3$ $\dot{x} = -6\sin\left(3t + \frac{\pi}{3}\right)$ $\ddot{x} = -6\cos\left(3t + \frac{\pi}{3}\right).\ 3$ $= -18\cos\left(3t + \frac{\pi}{3}\right)$ $\ddot{x} = -9\left(2\cos\left(3t + \frac{\pi}{3}\right)\right)$ $\ddot{x} = -3^2 x$ which is of the form $\ddot{x} = -n^2 x$ $\therefore$ particle is in SHM	2	1 mark for correct differentiations 1 mark for writing in the correct form and stating a conclusion
ii)	Period $= \frac{2\pi}{n}$ $= \frac{2\pi}{3}$	1	
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Question 7	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
a)(i)	If $\sin x - \sqrt{3}\cos x = R\sin(x - \alpha)$ then $R = \sqrt{1^2 + (\sqrt{3})^2}$ and $\tan\alpha = \sqrt{3}$ $\therefore R = 2$ and $\alpha = \frac{\pi}{3}$ So $\sin x - \sqrt{3}\cos x = 2\sin\left(x - \frac{\pi}{3}\right)$	2	1 mark for R and 1 mark for α
a) (ii)	$\sin x - \sqrt{3}\cos x = 1$ $2\sin\left(x - \frac{\pi}{3}\right) = 1$ $\sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$ $x - \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{\pi}{2}, \frac{7\pi}{6}$	2	1 for writing in form $\sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2}$ 1 for solving for x
b)	$x = 3t - 4$ ① $y = 2t^2 - t$ ② From ① $x + 4 = 3t$ $t = \frac{x+4}{3}$ Sub in ② $y = 2\left(\frac{x+4}{3}\right)^2 - \frac{x+4}{3}$ $y = \frac{2(x^2 + 8x + 16)}{9} - \frac{x+4}{3}$ $y = \frac{2x^2 + 16x + 32 - 3x - 12}{9}$ $y = \frac{2x^2 + 13x + 20}{9}$	3	1 for t subject 1 for subst into equation 2 1 for Cartesian equation
c) (i)	$N = 650 + Ae^{-kt}$ $\frac{dN}{dt} = -kAe^{-kt}$ $\frac{dN}{dt} = -k(N - 650)$	1	Need to differentiate and rearrange for the mark

Question 7	Trial HSC Examination - Mathematics Extension 1		2011
Part	**Solution**	**Marks**	**Comment**
c) (ii)	Let 2001 represent $t = 0$. $2400 = 650 + Ae^{0}$ $A = 2400 - 650$ $= 1750$ $N = 650 + 1750e^{-kt}$ When t = 4, N = 2000 $2000 = 650 + 1750e^{-4k}$ $\frac{1350}{1750} = e^{-4k}$ $-4k = ln\left(\frac{27}{35}\right)$ $k = -\frac{1}{4} ln\left(\frac{27}{35}\right)$ $k = 0.065$ (2 sig figs)	2	1 mark for *A*. 1 mark for *k*
c) (iii)	$N = 650 + 1\,750\, e^{-0.065(8)}$ $N = 1691$ There are 1691 wallabies in 2005.	1	1 for answer
c) (iv)	$1000 > 650 + 1\,750\, e^{-0.065t}$ $0.2 > e^{-0.065t}$ $-0.065t < ln(0.2)$ $t > \frac{ln(0.2)}{-0.065}$ $t > 24.8$ Population falls below 1000 after 25 years. (in the year 2026)	1	1 for answer as 24.8 years, 25 years or in the year 2026.
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