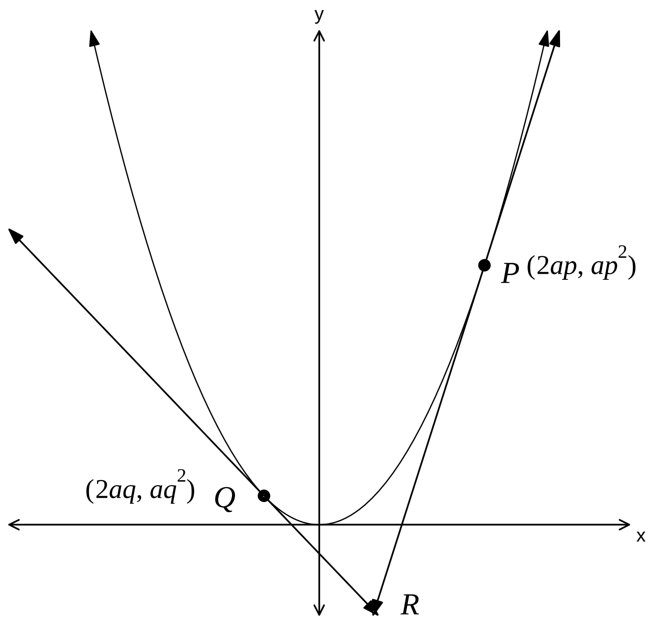
2010
TRIAL HSC
EXAMINATION

Mathematics
Extension 1

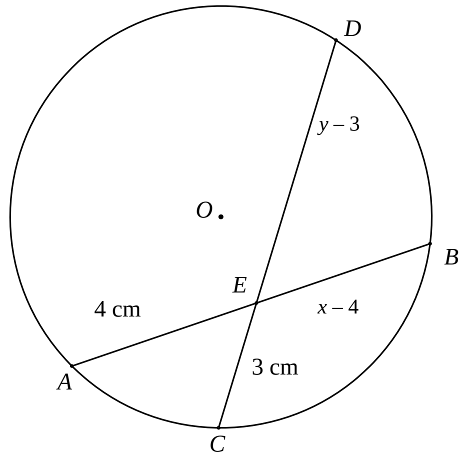
SOLUTIONS

Question 1	**Trial HSC Examination - Mathematics Extension 1**		**2010**
Part	**Solution**	**Marks**	**Comment**
a)	$x = \frac{kx_2 + lx_1}{k+l}$ $\quad$ $y = \frac{ky_2 + ly_1}{k+l}$ $3 = \frac{3x_2 + 2\times 6}{3+2}$ $\quad$ $2 = \frac{3y_2 + 2\times -1}{3+2}$ $15 = 3x_2 + 12$ $\quad$ $10 = 3y_2 - 2$ $3 = 3x_2$ $\quad$ $12 = 3y_2$ $x_2 = 1$ $\quad$ $y_2 = 4$ The point M is $(1,4)$	2	1 for correct substitution in formula 1 for point.
b)	$\int \frac{2}{\sqrt{x^2 + 25}}\,dx = 2\int \frac{1}{\sqrt{x^2+25}}\,dx$ $= \int \frac{1}{\sqrt{x^2 + 5^2}}\,dx$ $= 2\ln\left(x + \sqrt{x^2 + 25}\right) + C$ Using Standard Integrals.	2	1 for changing to standard form 1 for integral.
c)	$\frac{2x}{x-3} \leq 2$ $x \neq 3$ from denominator. $\frac{2x}{x-3}(x-3)^2 \leq 2(x-3)^2$ $2x(x-3) \leq 2x^2 - 12x + 18$ $2x^2 - 6x \leq 2x^2 - 12x + 18$ $6x \leq 18$ $x \leq 3$ But $x \neq 3$ So $x < 3$	3	1 for eliminating the denominator 1 for working solution to a linear equation 1 for solution
d)	$\sum_{r=1}^{5} r^2 + 2r = 1^2 + 2\times 1 + 2^2 + 2\times 2 + 3^2 + 2\times 3 + 4^2 + 2\times 4 + 5^2 + 2\times 5$ $= 3 + 8 + 15 + 24 + 35$ $= 85$	1	1 for answer

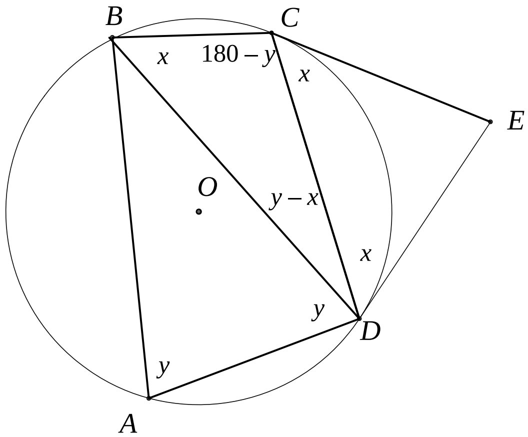
Question 1	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
e)	$P(x) = x^3 - 19x - 30$ $P(-3) = (-3)^3 - 19(-3) - 30$ $= -27 + 57 - 30$ $= 0$ $\therefore (x - -3) = (x + 3)$ is a factor of $P(x)$ Test $P(-5) = (-5)^3 - 19(-5) - 30$ $= -125 + 95 - 30 = -60$ $P(5) = (5)^3 - 19(5) - 30$ $= 125 - 95 - 30 = 0$ $\therefore (x - 5)$ is a factor of $P(x)$	2	1 for showing (x+3) is a factor 1 for finding either of other factors
f)	$u = x^2 - 2$ $du = 2x\,dx$ $\int \frac{x}{\sqrt{x^2 - 2}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2 - 2}}\,dx$ $= \frac{1}{2}\int \frac{du}{\sqrt{u}}$ $= \frac{1}{2}\int u^{-\frac{1}{2}}du$ $= u^{\frac{1}{2}} + C$ $= \sqrt{x^2 - 2} + C$	2	1 For correct method of substitution including du 1 for integral
		/12	

Question 2	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
a) (i)	 Equations of tangents: $y = px - ap^2$ and $y = qx - aq^2$ To find R, solve simultaneously $px - ap^2 = qx - aq^2$ $px - qx = ap^2 - aq^2$ $x(p - q) = a(p + q)(p - q)$ $x = a(p + q)$ $y = p \times a(p + q) - ap^2$ $= ap^2 + apq - ap^2$ $= apq$ R is the point $(a(p + q), apq)$	2	NB Graph not needed for marks. 1 for reasonable attempt to solve simultaneously 1 for correct result.
a) (ii)	Substitute $(0, a)$ into $y = \frac{p + q}{2}x - apq$ $a = \frac{p + q}{2} \times 0 - apq$ $-apq = a$ $pq = -1$	1 mark	

Question 2	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
a) (iii)	If chord passes through $(0, a)$ then $pq = -1$ and $p = -\frac{1}{q}$ R is the point $(a(p+q), apq)$. Which becomes $\left(a\left(-\frac{1}{q}+q\right), a \times (-1)\right)$ $x = a\left(-\frac{1}{q}+q\right)$ and $y = -a$ $x = -y\left(\frac{q^2-1}{q}\right)$ $qx = y(1-q^2)$ $y = \frac{qx}{1-q^2}$ OR SIMILARLY $y = \frac{px}{1-p^2}$	2	1 for introducing pq = -1 to eliminate p or q 1 for relating x and y and obtaining the equation of the locus.
b)	$\cos 2A = 1 - 2\sin^2 A$ $2\sin^2 A = 1 - \cos 2A$ $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ $\int \sin^2 6x\, dx = \frac{1}{2}\int 1 - \cos 12x\, dx$ $= \frac{1}{2}\left(x - \frac{1}{12}\sin 12x\right) + C$ $= \frac{x}{2} - \frac{\sin 12x}{24} + C$	2	1 for transformation of the integral or recalling a formula 1 for integration
c)	There are ${}^8\mathbf{P}_8$ (8!) ways the 8 vehicles can park. If the two utes are together, treat them as one, so there are 7 vehicles. These can park in ${}^7\mathbf{P}_7$ (7!) ways with ${}^2\mathbf{P}_2$ (2!) ways of arranging the utes among themselves. So the 7 are arranged in $\frac{7!}{2!}$ ways. Probability $= \frac{7!}{2!} \div 8!$ $= \frac{7!}{8!\,2!}$ $= \frac{1}{8 \times 2}$ $= \frac{1}{16}$	2	1 for arrangement of vehicles with utes together. 1 for probability

Question 2	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
d)	 Since $AB = x$, $EB = x - 4$ and since $CD = y$, $ED = y - 3$ $AE.EB = CE.ED$ (Ratio of intercepts on chords) $4(x-4) = 3(y-3)$ $4x - 16 = 3y - 9$ $4x = 3y + 7$	3	1 1 1
		/12	

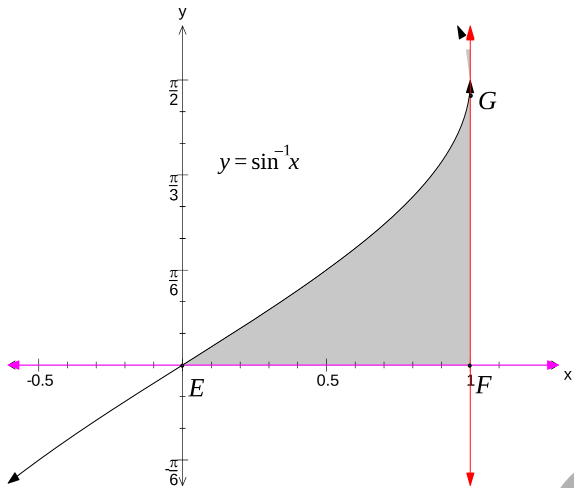
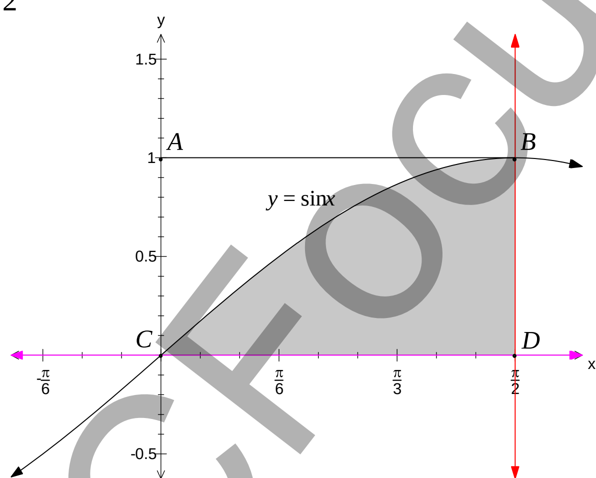
Question 3	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
a) (i)	$f(x) = (x^3 - 12x)^{\frac{1}{3}}$ $f(x) = \frac{1}{3}(x^3 - 12x)^{-\frac{2}{3}}.(3x^2 - 12)$	1	No need to simplify further
a) (ii)	$x_1 = -3.3$ $f(x_1) = ((-3.3)^3 - 12(-3.3))^{\frac{1}{3}}$ ≈ 1.54 $f'(x_1) = \frac{1}{3}((-3.3)^3 - 12(-3.3))^{-\frac{2}{3}} \times (3(-3.3)^2 - 12)$ ≈ 2.90 $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ $= -3.3 - \frac{1.54}{2.90}$ ≈ -3.83 (2 dec places)	2	1 for evaluating function and derivative. 1 for substitution into Newtons Method formula.
a) (iii)	As Newtons Method uses the intercept that the tangent makes, from the graph, the tangent at -3.3 is quite flat compared to the sudden drop in the curve to meet the axis. Hence the tangent would meet the axis much further along than the graph, so the second approximation is not as good as the first.	1	Mark for mention of the tangent meeting the axis or similar
b) (i)	$\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\sin\left(x + \frac{\pi}{4}\right) = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$ $= (\sin x) \times \frac{1}{\sqrt{2}} + (\cos x) \times \frac{1}{\sqrt{2}}$ $= \frac{\sin x + \cos x}{\sqrt{2}}$	2	1 correct definition 1 correct evaluation
b) (ii)	$\frac{\sin x + \cos x}{\sqrt{2}} = \frac{\sqrt{3}}{2}$ $\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$ $x + \frac{\pi}{4} = \frac{\pi}{3}$ or $\frac{2\pi}{3}$ $\left(\frac{\pi}{4} \le x + \frac{\pi}{4} \le \frac{9\pi}{4}\right)$ $x = \frac{\pi}{12}$ or $\frac{5\pi}{12}$ $(0 \le x \le 2\pi)$	2	1 for initial solution of $\frac{\pi}{3}$ and set. 1 for final solution for x

c) (i)	 $\angle EDC = \angle CBD = x$ (Angle between a tangent and a chord is equal to the angle in the alternate segment) Similarly $\angle ECD = \angle CBD = x$ Hence $\angle ECD = \angle EDC = x$ Or $EC = ED$ (Tangents from an external point are equal) Hence $\angle ECD = \angle EDC = x$ $\angle CED = 180 - \angle ECD - \angle EDC$ (Angle sum of triangle) $\angle CED = (180 - 2x)^\circ$	2	1 for partially completed proof with some of the required points or with single error 2 for completely correct proof Or any other valid proof
c) (ii)	$\angle BCD = 180 - y^\circ$ (Opposite angles of cyclic quadrilateral are supplementary) $\angle BDC = 180 - \angle CBD - \angle BCD$ $= 180 - x - (180 - y)$ $= 180 - x - 180 + y$ $= (y - x)^\circ$	2	1 for cyclic quad or similar partial proof. 2 for full proof.
		/12	

Question 4	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
a)	In expansion of $(a+b)^n$ $\quad T_{r+1} = {}^n\mathbf{C}_r a^{n-r} b^r$ $T_{r+1} = {}^9\mathbf{C}_r (x^2)^{9-r}\left(-\frac{2}{x}\right)^r$ $= {}^9\mathbf{C}_r x^{18-2r}(-2)^r x^{-r}$ In the expansion of $\left(x^2-\frac{2}{x}\right)^9$ $\quad = {}^9\mathbf{C}_r(-2)^r x^{18-3r}$ For the term independent of x, $18 - 3r = 0$ $3r = 18$ $r = 6$ $T_7 = {}^9\mathbf{C}_6(-2)^6 x^0$ $= 84 \times 64$ $= 5\,376$ Accept ${}^9\mathbf{C}_6(-2)^6$ for full marks.	3	1 for writing the general term or starting to write out the expansion. 1 for simplifying the term in x and setting to zero. 1 for term either as a single number or the unexpanded expression given.

b)	$P(x) = x^3 + mx^2 + 2mx + n$ $P(-3) = 0$ $(-3)^3 + m(-3)^2 + 2m(-3) + n = 0$ $3m + n = 27$ ① $P(2) = 0$ $(2)^3 + m(2)^2 + 2m(2) + n = 0$ $8m + n = -8$ ② $5m = -35$ ② – ① $m = -7$ $3(-7) + n = 27$ $n = 48$ $P(\alpha) = 0$ $\alpha^3 - 7\alpha^2 - 14\alpha + 48 = 0$ Three factors $(x-\alpha)(x-2)(x+3)$ give a constant term of 48 $-6(-\alpha) = 48$ $\alpha = 8$ OR Test factors, Try $P(8)$ $(8)^3 - 7(8)^2 - 14(8) + 48 = 0$ OR use division transformation ANSWER $m = -7$, $n = 48$, $\alpha = 8$	3	1 for setting up simultaneous equations 1 for solving for m and n 1 for evaluating α
c)	$f(x) = \log_e\left(\sqrt{9-x^2}\right)$, The log function has its argument greater than zero $\sqrt{9-x^2} > 0$ $9 - x^2 > 0$ $(3-x)(3+x) > 0$ Domain $-3 < x < 3$	2	1 for argument >0 1 for solving for the domain.
d) (i)	$P(Heavy, Heavy) = 0.05 \times 0.05$ $= 0.0025$	1	
d) (ii)	$P(3\ are\ heavy) = {}^{25}\mathbf{C}_3 (0.95)^{22} \times (0.05)^3$ $= 2300 \times (0.95)^{22} \times (0.05)^3$ $= 0.0930$ (3 sig fig)	1	

d) (iii)	$P(\text{at least 4 heavy}) =$ $1 - P(0\,H) - P(1\,H) - P(2\,H) - P(3\,H)$ $= 1 - {}^{25}\mathbf{C}_0\,(0.95)^{25} \times (0.05)^0 - {}^{25}\mathbf{C}_1(0.95)^{24} \times (0.05)^1$ $= {}^{-25}\mathbf{C}_2(0.95)^{23} \times (0.05)^2 - {}^{25}\mathbf{C}_3(0.95)^{22} \times (0.05)^3$ $= 1 - 0.2774 - 0.3650 - 0.2305 - 0.0930$ $= 0.034$ (2 sig fig)	2	1 for identifying complement and terms required 1 for answer
		/12	

Question 5	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
a) (i)	The area enclosed between $y = \sin^{-1} x$, the x axis, and the line $x = 1$  The area enclosed between $y = \sin x$, the x axis, and the line $x = \frac{\pi}{2}$. 	2	1 for sketch graph 1 for sketch graph

Question 5	**Trial HSC Examination - Mathematics Extension 1**		**2010**
Part	**Solution**	**Marks**	**Comment**
a) (ii)	$\int_0^1 \sin^{-1} x\, dx$ corresponds to the area *EFG*, which is congruent to the area ABC by symmetry of inverse functions about $y = x$. Together with area BCD which corresponds to $\int_0^{\frac{\pi}{2}} \sin x\, dx.$ they form the rectangle *ABCD* which has area $A = 1 \times \frac{\pi}{2} = \frac{\pi}{2}$ So $\int_0^1 \sin^{-1} x\, dx + \int_0^{\frac{\pi}{2}} \sin x\, dx = \frac{\pi}{2}$ And hence $\int_0^1 \sin^{-1} x\, dx = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \sin x\, dx.$	2	1 mark for a partial explanation that relates the areas of the two functions 2 marks for a fuller explanation that clearly relates the areas to the rectangle and explains the required relationship
b) (i)	$(1+x)^{n-1} = \binom{n-1}{0} 1^{n-1} + \binom{n-1}{1} 1^{n-2} x + \binom{n-1}{2} 1^{n-3} x^2 + \dots .$ $\dots + \binom{n-1}{n-2} 1^1 x^{n-2} + \binom{n-1}{n-1} x^{n-1}$ Let $x = 1$ $(1+1)^{n-1} = \binom{n-1}{0} 1^{n-1} + \binom{n-1}{1} 1^{n-2} (1) + \binom{n-1}{2} 1^{n-3} (1)^2 + \dots$ $\dots . + \binom{n-1}{n-2} 1^1 (1)^{n-2} + \binom{n-1}{n-1} (1)^{n-1}$ $\binom{n-1}{1} 1^{n-2} (1) + \binom{n-1}{2} 1^{n-3} (1)^2 + \dots$ $\dots . + \binom{n-1}{n-2} 1^1 (1)^{n-2} + 2 = 2^{n-1}$ $\binom{n-1}{1} + \binom{n-1}{2} + \dots$ $\dots . + \binom{n-1}{n-2} = 2^{n-1} - 2$	2	1 for the expansion of $(1+x)^n$ 1 for obtaining required result

Question 5	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
b) (ii)	$2^{n-1} - 2 > 1000$ $2^{n-1} > 1002$ $(n-1)\ln 2 > \ln 1002$ $n - 1 > \frac{\ln 1002}{\ln 2}$ $n - 1 > 9.9$ $n > 10.9$ Least positve integer, $n = 11$	2	1 for using logs or trial and error 1 for answer
c) (i)	Endpoints where $v = 0$ $v^2 = 24 + 2x - x^2 = 0$ $(6-x)(4+x) = 0$ $x = 6$ and $x = -4$	1	1 mark for two endpoints
c) (ii)	$v^2 = 24 + 2x - x^2 = 0$ Acceleration $= \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $= \frac{d}{dx}\left(\frac{24 + 2x - x^2}{2}\right)$ $= \frac{2 - 2x}{2}$ Acceleration $= 1 - x$	2	1 mark for a $= \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ 1 mark for answer
c) (iii)	$a = -n^2 x$ $\ddot{x} = 1 - x$ $\ddot{x} = -1^2(x-1)$ $n = 1$ Period $T = \frac{2\pi}{n}$ Period $T = 2\pi$ seconds	1	1 mark for answer
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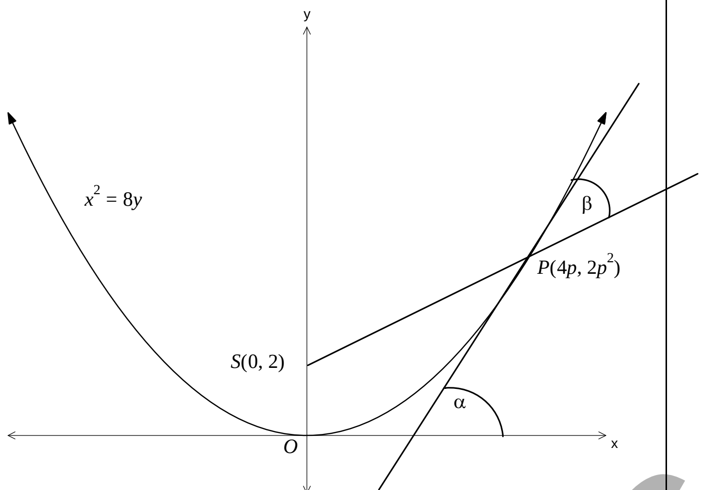
Question 6	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
a) (i)	 $y = \frac{x^2}{8}$ $\frac{dy}{dx} = \frac{2x}{8} = \frac{x}{4}$ At $P(4p, 2p^2)$ $\frac{dy}{dx} = \frac{4p}{4} = p$	1	1 mark for using derivative
(ii)	Grad SP $= \frac{2p^2 - 2}{4p - 0}$ $= \frac{p^2 - 1}{2p}$	1	1 mark for using gradient form correctly

Question 6	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
(iii)	$\tan\beta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$ $= \dfrac{p - \dfrac{p^2 - 1}{2p}}{1 + p\left(\dfrac{p^2 - 1}{2p}\right)}$ $= \dfrac{\dfrac{2p^2 - p^2 + 1}{2p}}{\dfrac{2 + p^2 - 1}{2}}$ $= \dfrac{\dfrac{p^2 + 1}{2p}}{\dfrac{p^2 + 1}{2}}$ $= \dfrac{p^2 + 1}{2p} \times \dfrac{2}{p^2 + 1}$ $= \dfrac{1}{p}$	2	1 for formula for angle between two lines. 1 for simplifying Or any other valid method
(iv)	$\tan\alpha = p$ $\tan\beta = \dfrac{1}{p} = \dfrac{1}{\tan\alpha} = \cot\alpha$ $\cot\alpha = \tan\left(\dfrac{\pi}{2} - \alpha\right)$ So $\tan\beta = \tan\left(\dfrac{\pi}{2} - \alpha\right)$ $\beta = \dfrac{\pi}{2} - \alpha$ $\alpha + \beta = \dfrac{\pi}{2}$	1	1 mark for any reasonable relating of α and β to get required result.

Question 6	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
b)	Show true for $n=1$ $4^1+14=14+4$ $=18=6\times 3$ $\therefore$ true for $n=1$ Assume true for $n=k$ $4^k+14=6p$ Consider $n=k+1$ $4^{k+1}+14=4\times 4^k+14$ $=4\times 4^k+4\times 14-3\times 14$ $=4(4^k+14)-3\times 14$ $=4\times 6p-6\times 7$ $=6(4p-7)$ $\therefore 4^k+14$ is divisible by 6 Hence if proposition is true for $n=k$ it is true for $n=k+1$ But since true for $n=1$ by induction is true for all $n\geq 1$	3	1 mark for showing true for $n = 1$ 1 mark for showing case for $n = k + 1$ 1 mark if all necessary steps for induction are done, including the statement of assumption and the conclusion
c)	$f(x) = 4x^2 - 5x$ $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4(x+h)^2 - 5(x+h) - 4x^2 + 5x}{h}$ $= \dfrac{4(x^2+2xh+h^2) - 5(x+h) - 4x^2}{h}$ $= \dfrac{4x^2 + 8xh + 4h^2 - 5x - 5h - 4x^2 +}{h}$ $= \dfrac{8xh + 4h^2 - 5h}{h}$ $= \dfrac{h(8x+4h-5)}{h}$ $f'(x) = \lim_{h\to 0} \dfrac{h(8x+4h-5)}{h}$ $= \lim_{h\to 0} (8x+4h-5)$ $= 8x - 5$	2	1 mark for substitution into formula and simplifying 1 mark for eliminating h and obtaining required result

Question 6	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
d)	$\frac{d}{dx}(2x^2\cos^{-1}2x) = 2x^2\frac{-1}{\sqrt{\frac{1}{4}-x^2}} + 4x\cos^{-1}2x$ $= 2x^2\frac{-2}{\sqrt{1-4x^2}} + 4x\cos^{-1}2x$ $= \frac{-4x^2}{\sqrt{1-4x^2}} + 4x\cos^{-1}2x$	2	1 mark for use of product rule 1 mark for completing derivatives and simplifying (2nd last line or equivalent is acceptable as the answer)
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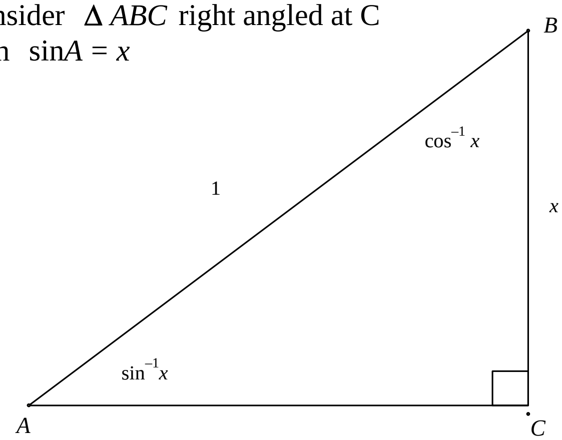
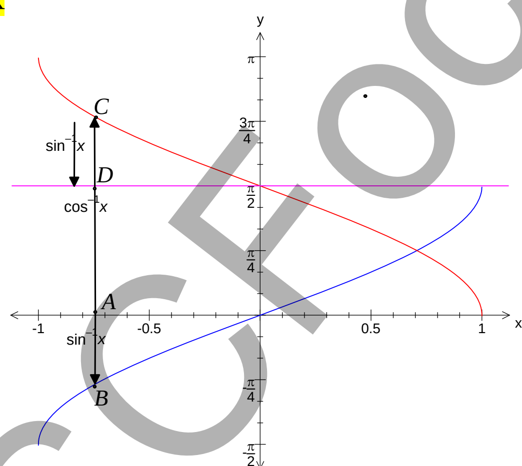
Question 7	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
(a)	Consider ΔABC right angled at C with $\sin A = x$  $\sin A = x \Rightarrow \angle A = \sin^{-1} x$ $\cos B = x \Rightarrow \angle B = \cos^{-1} x$ $\angle A + \angle B = \frac{\pi}{2}$ (angle sum of right triangle) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ OR  $AB = \sin^{-1} x$ $CD = \sin^{-1} x$ (By symmetry of graphs) $AC = \cos^{-1} x$ $DA = \frac{\pi}{2}$ $AC + CD = AD$ $\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$	2	1 mark for graph or diagram 1 mark for explanation Other possible explanations and graphs are acceptable.

Question 7	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
b) (i)	$\ddot{x} = -x$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -x \quad (v = \dot{x})$ $\frac{1}{2}v^2 = -\frac{x^2}{2} + C$ At $x = 0$, $\dot{x} = v = 1$ $\frac{1}{2}(1)^2 = 0 + C$ $C = \frac{1}{2}$ $\frac{1}{2}v^2 = -\frac{x^2}{2} + \frac{1}{2}$ $v^2 = 1 - x^2$ $\lvert\dot{x}\rvert = \sqrt{1 - x^2}$	2	1 mark for using $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$ Or other way of relating v in terms of x 1 mark for working to required result
b) (ii)	Let $x = a\cos(nt + \alpha)$ $a = 1$ and $n = 1$ $x = \cos(t + \alpha)$ $x = 0$, when $t = 0$ $0 = \cos\alpha$ $\alpha = \frac{\pi}{2}$ $x = \cos\left(t + \frac{\pi}{2}\right)$ Let $x = a\sin(nt + \alpha)$ $a = 1$ and $n = 1$ $x = \sin(t + \alpha)$ $x = 0$, when $t = 0$ $0 = \sin\alpha$ $\alpha = 0$ $x = \sin(t)$	2	1 for form of equation either sin or cos. 1 for correct values of constants

Question 7	Trial HSC Examination - Mathematics Extension 1		2010
Part	**Solution**	**Marks**	**Comment**
c) (i)	$\frac{dV}{dt} = 0.5$ $V = 0.5t + C$ When $t = 0$, $V = 0$ $V = \frac{t}{2}$ $V = \frac{4}{3}\pi r^3$ $\frac{t}{2} = \frac{4}{3}\pi r^3$ $3t = 8\pi r^3$ $r^3 = \frac{3t}{8\pi}$ $r = \sqrt[3]{\frac{3t}{8\pi}}$	2	1 1

Question 7	Trial HSC Examination - Mathematics Extension 1	2010

Part	Solution	Marks	Comment
c) (ii)	$r = \sqrt[3]{\frac{3t}{8\pi}}$ $r = \left(\frac{3t}{8\pi}\right)^{\frac{1}{3}}$ $r = \left(\frac{3}{8\pi}\right)^{\frac{1}{3}} \times t^{\frac{1}{3}}$ $\frac{dr}{dt} = \left(\frac{3}{8\pi}\right)^{\frac{1}{3}} \times \frac{t^{-\frac{2}{3}}}{3}$ $A = 4\pi r^2$ $\frac{dA}{dr} = 8\pi r$ $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$ $= 8\pi\, r \times \left(\frac{3}{8\pi}\right)^{\frac{1}{3}} \times \frac{t^{-\frac{2}{3}}}{3}$ $= 8\,\pi \left(\left(\frac{3t}{8\pi}\right)^{\frac{1}{3}}\right) \times \left(\left(\frac{3}{8\pi}\right)^{\frac{1}{3}} \times \frac{t^{-\frac{2}{3}}}{3}\right)$ When $t = 8$ $\frac{dA}{dt} = 8\,\pi \left(\left(3\frac{8}{8\pi}\right)^{\frac{1}{3}}\right) \times \left(\left(\frac{3}{8\pi}\right)^{\frac{1}{3}} \times \frac{(8)^{-\frac{2}{3}}}{3}\right)$ $= 8\,\pi \times \left(\frac{3}{\pi}\right)^{\frac{1}{3}} \times \frac{1}{2}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} \times \frac{1}{12}$ $= \frac{\pi}{3}\left(\frac{3}{\pi}\right)^{\frac{2}{3}}$ $= \frac{\pi}{3}\left(\frac{\pi}{3}\right)^{-\frac{2}{3}}$ $= \left(\frac{\pi}{3}\right)^{\frac{1}{3}}$ $= \sqrt[3]{\frac{\pi}{3}}$	3	1 mark for obtaining $\frac{dr}{dt}$ 1 mark for expression for $\frac{dA}{dt}$ 1 mark for evaluating this when $t = 8$

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	OR $r=\sqrt[3]{\frac{3t}{8\pi}}$ $r=\left(\frac{3t}{8\pi}\right)^{\frac{1}{3}}$ $\frac{dr}{dt}=\frac{1}{3}\times\frac{3}{8\pi}\times\left(\frac{3t}{8\pi}\right)^{-\frac{2}{3}}$ $=\frac{1}{8\pi}\left(\frac{3t}{8\pi}\right)^{-\frac{2}{3}}$ $A=4\pi r^2$ $\frac{dA}{dr}=8\pi r$ $\frac{dA}{dt}=\frac{dA}{dr}\times\frac{dr}{dt}$ $\frac{dA}{dt}=8\pi r\times\frac{1}{8\pi}\left(\frac{3t}{8\pi}\right)^{-\frac{2}{3}}$ $=r\times\left(\frac{3t}{8\pi}\right)^{-\frac{2}{3}}$ $=\left(\frac{3t}{8\pi}\right)^{\frac{1}{3}}\times\left(\frac{3t}{8\pi}\right)^{-\frac{2}{3}}$ $=\left(\frac{3t}{8\pi}\right)^{-\frac{1}{3}}$ $when\ \ t=8$ $\frac{dA}{dt}=\left(\frac{24}{8\pi}\right)^{-\frac{1}{3}}$ $=\left(\frac{3}{\pi}\right)^{-\frac{1}{3}}$ $=\left(\frac{\pi}{3}\right)^{\frac{1}{3}}$		

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Part	**Solution**	**Marks**	**Comment**
c) (iii)	$4\pi r^2 = 200$ $r^2 = \frac{50}{\pi}$ $r = \sqrt{\frac{50}{\pi}}$ $r = \sqrt[3]{\frac{3t}{8\pi}}$ $\sqrt{\frac{50}{\pi}} = \sqrt[3]{\frac{3t}{8\pi}}$ $\left(\sqrt{\frac{50}{\pi}}\right)^6 = \left(\sqrt[3]{\frac{3t}{8\pi}}\right)^6$ $\frac{125000}{\pi^3} = \frac{9t^2}{64\pi^2}$ $t^2 = \frac{125000}{\pi^3} \cdot \frac{64\pi^2}{9}$ $= \frac{8000000}{9\pi}$ $= 282\ 942$ $t = 532$ seconds $t = 8$ minutes and 52 seconds	1	1 mark for answer in seconds or minutes and seconds or in unsimplified form.
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