2010

SEMESTER 1
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Student Number

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Answer questions in writing booklets provided
- Start a new page for each question
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 75

- Attempt questions 1-5
- All questions are of equal value

Question 1 (15 marks) Start a new sheet of writing paper. Marks

(a) Given the complex numbers $z = 3+4i$ and $\omega = 2-3i$, write the following in the form $x+iy$

(i) $z+\omega$ **1**

(ii) $\dfrac{z}{\omega}$ **1**

(iii) $\sqrt{z}$ **3**

(b) If the complex number $z = 1+\sqrt{3}i$

(i) Find $|z|$ and arg z **2**

(ii) Hence write z in modulus argument form **1**

(iii) By using your answer for part (ii) or otherwise, write the complex number z^4 in the form $x+iy$. **1**

(c) The triangle POQ is right angled at O. The length of OQ is twice that of OP. (O is the origin and Q is in the second quadrant). Given that OP represents the complex number $3+4i$

(i) Determine the complex number represented by OQ **1**

(ii) Determine the complex number represented by QP **1**

(d) Given that $z = \cos\theta + i\sin\theta$

(i) Show that $z^n + z^{-n} = 2\cos n\theta$ **1**

(ii) hence by using (i) and binomial expansion, write $\cos^4\theta$ in terms of $\cos 2\theta$ and $\cos 4\theta$ **3**

End of Question 1

Question 2 (15 marks) Start a new sheet of writing paper. **Marks**

(a) Sketch the graph of the parabola $f(x) = x^2 - x - 2$ **1**

Use this to draw sketches of the following functions indicating particularly any asymptotes and intercepts with the axes.

(i) $y = |f(x)|$ **2**

(ii) $y = [f(x)]^2$ **2**

(iii) $y^2 = f(x)$ **2**

(iv) $\dfrac{1}{f(x)}$ **2**

(v) $y = \log[f(x)]$ **2**

(b) Find the minimum value of $x^2 \log x$ and sketch the graph of $y = x^2 \log x$

$[note : x > 0]$ **4**

End of Question 2

Question 3 (15 marks) Start a new sheet of writing paper. **Marks**

a) Find the value of k for which $(x-1)$ is a factor of the polynomial $p(x) = x^{11} - 3x^6 + kx^4 + x^2$ **1**

(b) The equation $x^3 + x^2 + 2 = 0$ has roots α, β, γ. **5**

Evaluate

(i) $\alpha + \beta + \gamma$

(ii) $\alpha\beta + \beta\gamma + \alpha\gamma$

(iii) $\alpha\beta\gamma$

(iv) $\alpha^2 + \beta^2 + \gamma^2$

(v) $\alpha^3 + \beta^3 + \gamma^3$

Also write down the polynomial equation which has roots $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$. **2**

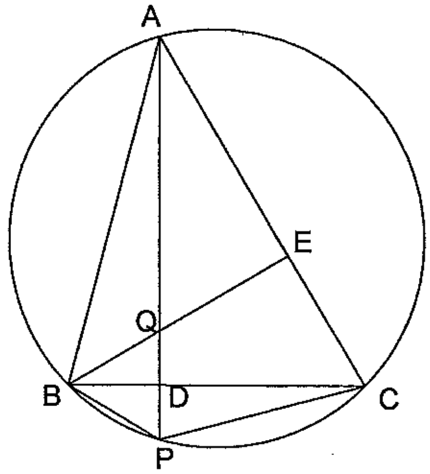
(c) Find the values of the real numbers p and q if $x^2 + 1$ is a factor of the polynomial $p(x) = x^4 + px^3 + 2x + q$. Hence factorise $p(x)$ over R (real field) and over C (complex field). **3**

(d) Find all the roots of $p(x) = 18x^3 + 3x^2 - 28x + 12 = 0$ given that two of the roots are equal. **4**

End of Question 3

Question 4 (15 marks) **Start a new sheet of writing paper.** **Marks**

(a)



ABC is an acute angled triangle inscribed in a circle. D is the point on BC such that AD is perpendicular to BC. AD produced meets the circle at P. Q is the point on AD such that DQ=DP.
BQ produced meets CA at E.

(i) Copy the diagram showing the above information **1**

(ii) Show that $\Delta BDP \equiv \Delta BDQ$ **3**

(iii) Show that $BDEA$ is a cyclic quadrilateral **4**

(iv) Show that BE is perpendicular to CA **2**

(b) (i) Show that : $\sin x + \sin 3x = 2\sin 2x \cos x$ **2**

(ii) Hence, or otherwise, find all solutions of

$$\sin x + \sin 2x + \sin 3x = 0 \text{, for } 0 \le x \le 2\pi$$ **3**

End of Question 4

Question 5 (15 marks) **Start a new sheet of writing paper.** **Marks**

(a) (i) On an Argand diagram shade in the region **R** containing all points representing complex numbers z such that

$$1 < |z| < 2 \quad \text{and} \quad \frac{\pi}{4} < \arg z < \frac{\pi}{2}$$ **2**

(ii) In **R,** mark with a dot a point **K** representing a complex number z. Clearly indicate on your diagram the points **M,N,P** and **Q** representing the complex numbers $\bar{z}, -z, \frac{1}{z}$ and $2z$ respectively. **4**

(b) Show that the locus specified by $3|z-(4+i)| = |z-(12+12i)|$ is a circle. Write down radius and the coordinates of its centre. Draw a neat sketch of the circle on the Argand diagram. **4**

(c) (i) Show the roots of $z^5+1=0$ on a unit circle in an Argand diagram. **3**

(ii) Factor z^5+1 into irreducible factors with real coefficients **2**

End of Question 5

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$