# HORNSBY GIRLS HIGH SCHOOL

# 2010
# TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Extension 1



**General Instructions**

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown for every question
- Begin each question in a new booklet

**Total marks (84)**

- Attempt Questions 1 – 7
- All questions are of equal value

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**Total Marks**
**Attempt Questions 1–7**
**All Questions are of equal value**

Begin each question in a new booklet, writing your student number and question number in the boxes indicated. Extra writing booklets are available.

**Question 1** (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The interval $AB$, where $A$ is $(-3, 4)$ and $B$ is $(1, -2)$, is divided **externally** in the ratio 1: 3 by the point $P\ (x,\ y)$. Find the values of $x$ and $y$. **2**

(b) Differentiate $\cos^{-1}(x^3)$ with respect to $x$. **2**

(c) Prove that, if $x^4 - x^3 + kx - 4$ has a factor of $(x+1)$, then it also has a factor of $(x-2)$. **2**

(d) Solve $\dfrac{x+4}{x-2} \geq 3$ for $x$. **3**

(e) Use the substitution $u = 2x+1$ to evaluate $\displaystyle\int_0^1 \frac{4x}{2x+1}\,dx$. **3**

**Question 2** (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve (giving your answer to nearest degree): **3**

$$3\cos x + 5\sin x = 5 \text{ for } 0° \le x \le 360°$$

(b) The curves $y = x^2$ and $y = 2x$ meet at $x = 2$. Find the angle between these curves at this point of intersection. **2**

(c) Find the primitive function of $2\sin^2 x$. **2**

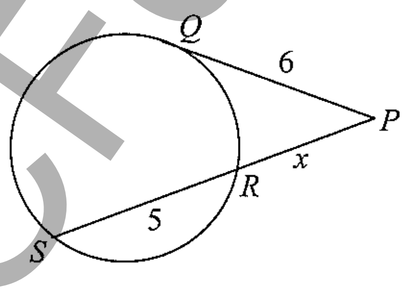
(d) In a large school, 5% of the students have blond hair. A group of 10 students is randomly chosen.

What is the probability that:

(i) exactly one student has blond hair? **1**

(ii) at least 2 students have blond hair? **2**

(e)




PQ is a tangent to a circle $QRS$, while $PRS$ is a secant intersecting the circle at $R$ and $S$, as shown in the diagram. **2**

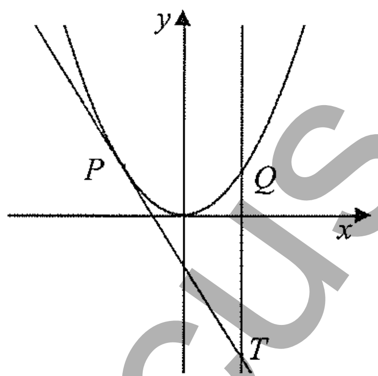
Given that $PQ = 6$, $RS = 5$ and $PR = x$, find the value of $x$.

**Question 3** (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the volume of the solid of revolution formed when the region bounded by the curve $y = \dfrac{1}{\sqrt{4+x^2}}$, the $x$-axis, the $y$-axis and the equation $x = \dfrac{\pi}{2}$ is rotated about the $x$-axis. **3**

(b) Find the term independent of $x$ in the expansion of $\left(3x^2 + \dfrac{2}{x}\right)^6$. **3**

(c)




Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

The tangent at P and the line through Q parallel to the $y$ axis intersect at point $T$.

The equation of the tangent at $P$ is $y = px - ap^2$. **(Do NOT prove this)**

(i) Find the coordinates of $T$. **1**

(ii) Write down the coordinates of $M$, the midpoint of $PT$. **1**

(iii) Determine the locus of $M$ when $pq = -1$. **1**

(d) A sphere is being heated so that its surface area is expanding at a constant rate of $0.025\ \text{cm}^2$ per second. **3**

Find the rate of change of the volume when the radius is 5 cm.

**Question 4** (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the function $f(x) = \dfrac{2x}{1-x^2}$.

(i) Show that the function is increasing for all values of $x$ in its domain. **2**

(ii) Sketch the graph of $y = f(x)$ showing the intercepts on the axes and any asymptotes. **3**

(iii) Hence, or otherwise, find the values of $k$ such that $\dfrac{2x}{1-x^2} = k$ has two solutions. **1**

(b) Evaluate $\lim\limits_{x\to 0} \dfrac{1-\cos^2 2x}{x^2}$. **2**

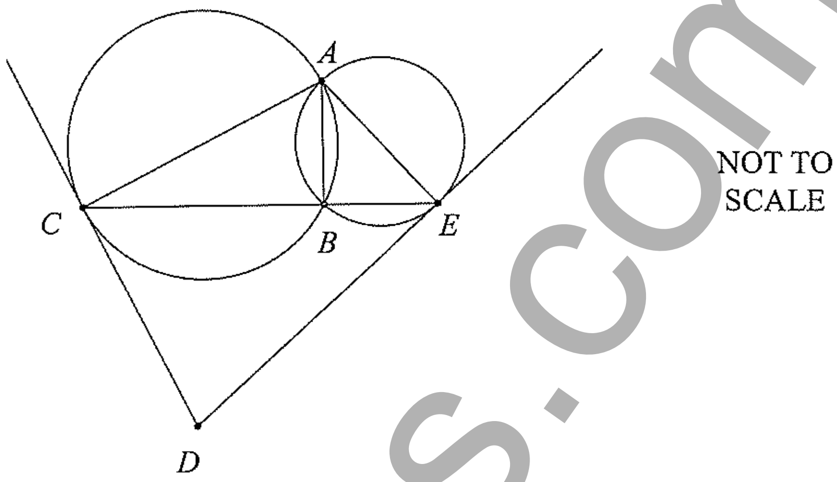
(c) Peter and his brother James are having a card night for themselves and 6 other friends. If they are to be seated at a round table, what is the probability that Peter and James do NOT sit next to each other? **2**

(d) Find the maximum value of $2x(1-x)$ and hence determine the range of $y = \sin^{-1}\left[2x(1-x)\right]$ for $0 \le x \le 1$. **2**

**Question 5** (12 marks) Use a SEPERATE writing booklet. **Marks**

(a) Use mathematical induction to prove that $4^n > 2n+1$, where $n$ is a positive integer. **3**

(b)




In the diagram above, two circles intersect at A and B. **3**

Points C and E lie on the circles and C, B and E are collinear.

Tangents at C and E meet at D.

Show that quadrilateral AEDC is concyclic.

(c) The population, $P$, of a mining town after $t$ years satisfies the equation

$$\frac{dP}{dt} = k(P-1000).$$

The population was initially 10 000, and after five years it had decreased to 8 000.

(i) Show that $P = 1000 + Ae^{kt}$ is a solution of the equation. **1**

(ii) Find the value of $A$. **1**

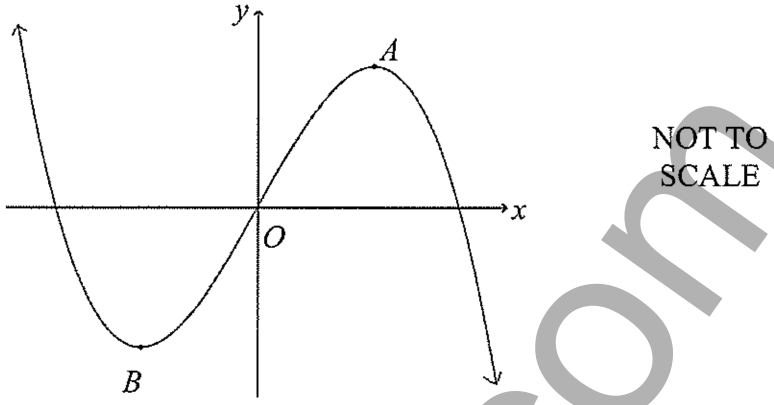
(iii) Find the value of $k$. (Give your answer correct to 3 significant figures) **1**

(iv) Find the number of years taken for the population to reach 5000. **2**

(v) Sketch the graph of the population against time. **1**

**Question 6** (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The graph of $y = f(x)$, where $f(x) = 3x - x^3$, is shown in the diagram below.




(i) Find the coordinates of the turning points $A$ and $B$. **2**

(ii) Find the largest domain containing the origin for which $f(x)$ has an inverse function. **1**

(iii) By considering the graph of $y = f(x)$, find the domain of $f^{-1}(x)$. **1**

(iv) By considering the gradient of $y = f(x)$, or otherwise, find the gradient of the inverse function $y = f^{-1}(x)$ at $x = 0$. **2**

(b) A particle moves in a straight line, so that its acceleration $x$ cm from the origin is given by $\frac{d^2x}{dt^2} = 15 - 25x$. Initially the particle is at rest 1.6 cm to the right of the origin.

(i) Show that the speed is given by $s = \sqrt{30x - 25x^2 + 16}$. **2**

(ii) Given that the motion is simple harmonic, find the interval in which the particle moves. **2**

(iii) Find the maximum speed, and the displacement where this occurs. **2**

**Question 7** (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Two of the roots of the equation $x^3 - ax^2 + b = 0$ are reciprocals.

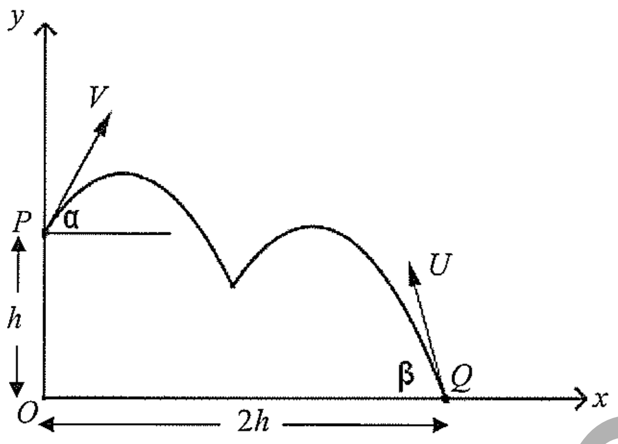
(i) Show that the third root is equal to $-b$. **2**

(ii) Show that $a = \frac{1}{b} - b$. **2**

(iii) Show that the 2 roots, which are reciprocals, will be real if $\frac{-1}{2} \leq b \leq \frac{1}{2}$. **2**

**Question 7 continues on page 10**

(b)




$O$ and $Q$ are two points $2h$ metres apart on horizontal ground. $P$ is a point $h$ metres directly above $O$. Particle $A$ is projected from $P$ towards $Q$ with speed $V$ m/s at an angle $\alpha$ above the horizontal.

At the same time particle $B$ is projected from $Q$ towards $P$ with speed $U$ m/s at an angle $\beta$ above the horizontal. The two particles collide $T$ seconds after projection.

For particle $A$ the equations of motion are: $\ddot{x}_p = 0$ and $\ddot{y}_p = -g$.

It is known that its horizontal distance $x_P$ from $O$, is given by:

$x_P = Vt\cos\alpha$ , where $t$ is time in seconds. **(Do NOT prove this)**

(i) Use calculus to show that at time $t$ seconds, its vertical distance $y_P$ from $O$ is given by: **2**

$$y_P = Vt\sin\alpha - \tfrac{1}{2}gt^2 + h$$

(ii) For particle $B$, write down expressions for its horizontal distance $x_Q$ from $Q$ and its vertical distance $y_Q$ from $Q$ at time t seconds. **2**

(iii) By considering the point of collision, find an expression for $\dfrac{V}{U}$ in terms of $\alpha$ and $\beta$. **2**

**End of paper**