Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Afternoon Session
Thursday, 20 August 2009

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value

6300 - 1

Total marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Find the remainder when $P(x) = x^3 - 3x^2 + 3x - 5$ is divided by $x - 2$. **2**

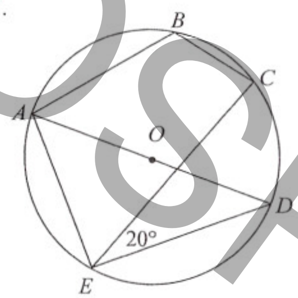
(b) Find $\int \sin^2 6x \, dx$. **2**

(c) Sketch the graph of $y = 3\sin^{-1}(2x)$, clearly indicating the domain and range. **3**

(d) (i) Find the Cartesian equation of the curve with parametric equations $x = \cos t$ and $y = 3 + \sin t$. **2**

(ii) Describe this locus geometrically. **1**

(e) In the diagram AOD and EC are straight lines, O is the centre of the circle, and $\angle CED = 20^\circ$. **2**



NOT TO SCALE

Find $\angle ABC$, giving reasons for your answer.

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\lim_{x \to 0} \frac{\sin 3x}{x}$. **1**

(b) Use the substitution $u = 3x - 1$ to evaluate $\int_1^2 \frac{x}{3x-1}\,dx$. **3**

(c) Find all real numbers such that $\ln(2x+3) + \ln(x-2) = 2\ln(x+4)$. **4**

(d) (i) From a group of 7 girls and 6 boys, 3 girls and 2 boys are chosen. How many different groups of 5 are possible? **2**

(ii) If the group of 5 stands in a line what is the probability that the boys stand together? **2**

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve the inequality $\frac{x^2-4}{x+3} < x - 4$ for x. **3**

(b) Prove by Mathematical Induction that $3^{3n} + 2^{n+2}$ is divisible by 5, for all positive integers n. **4**

(c) A particle, P, moves on the x-axis for time $t \geq 0$, in seconds, with velocity $v = \frac{2}{1+3x}$ cms^{-1}, where x, in centimetres, is the displacement from the origin $x = 0$.

(i) Find an expression for the acceleration, a cms^{-2}, and show that a varies directly with v^3. **3**

(ii) If the particle was initially at the origin, describe the motion both initially and as $t \to \infty$. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) The function $f(x) = e^x - x - 2$ has a zero near $x = 1.2$. **2**
Use one application of Newton's method to find a second approximation to the zero. Write your answer correct to three significant figures.

(b) A function is defined by $f(x) = e^{3x} - 1$ for all real x.

(i) Draw the graph of $y = f(x)$ and state the range of the function. **2**

(ii) Find the inverse function, $f^{-1}(x)$, clearly indicating any restrictions. **3**

(c) A particle moves in a straight line so that its displacement x cm from the origin at time $t \geq 0$, in seconds, is given by $x = \sqrt{3}\cos 3t - \sin 3t$.

(i) Show that the particle moves in simple harmonic motion. **2**

(ii) Find the velocity when the particle is 1 cm from the origin on its first oscillation. **3**

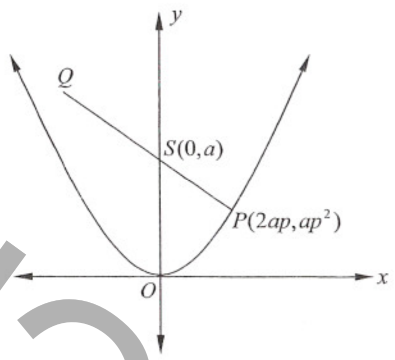
Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) If the roots of $x^3 - 6x^2 + 3x + k = 0$ are consecutive terms of an arithmetic series show that one of the roots is 2. **2**

(ii) Hence find the value of k and the other two roots. **3**

(b) Show that $\dfrac{\tan 2\theta - \tan\theta}{\tan 2\theta + \cot\theta} = \tan^2\theta$. **3**

(c) $P(2ap,\ ap^2)$ is a point on the parabola $x^2 = 4ay$ with focus $S(0,\ a)$. The point Q lies on PS produced and Q divides PS so that $PQ : QS = -4 : 3$.



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(i) Show that Q has coordinates $\left(-6ap,\ a(4-3p^2)\right)$. **2**

(ii) Show that as P varies, the locus of Q is a parabola. **2**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) Simplify $\dfrac{2^{4n} \times 3^{2n}}{8^n \times 6^n} + 3^n$. **3**

(b) A balloon in the shape of a cylinder, with height h and radius r, expands so that h is always proportional to r, that is $h = kr$ for some constant k.

When $r = 4\,\text{cm}$, the volume is expanding at the rate of $0.2\,\text{cm}^3\,\text{s}^{-1}$.

(i) Show that when $r = 4\,\text{cm}$ the rate of change of the radius is given by $\dfrac{dr}{dt} = \dfrac{1}{240\pi k}$. **2**

(ii) If the surface area of the balloon is expanding at the rate of $0.1\,\text{cm}^2\,\text{s}^{-1}$ when $r = 4\,\text{cm}$, find the constant of proportionality, k. **3**

(c) (i) Differentiate both sides of the expansion $(1+x)^{2n} = \sum_{k=0}^{2n} {}^{2n}C_k\, x^k$. **2**

(ii) Hence show that $\sum_{k=1}^{2n} k\, {}^{2n}C_k = n \times 4^n$. **2**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

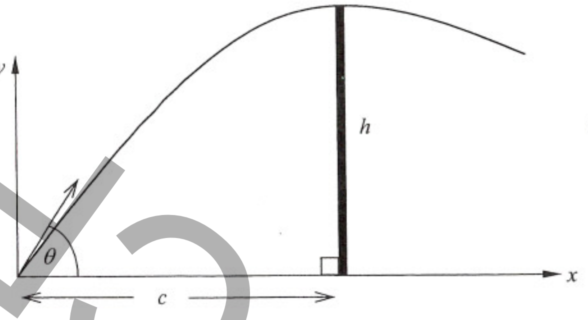
(a) A student is taking a test with 50 multiple-choice questions and guesses the answer to each one. The probability of guessing a question correctly is 0.3.

(i) What is the probability that the student answers 25 questions correctly? **2**

(ii) What is the most likely number of questions answered correctly? **3**

(b) A vertical wall, height h metres, stands on horizontal ground. When a projectile is fired, in a vertical plane which is at right angles to the wall, from a point on the ground c metres from the wall, it just clears the wall at the highest point of its path. The equations of motion for the projectile with angle of projection, θ, are:

$x = Vt\cos\theta \qquad y = Vt\sin\theta - \frac{1}{2}gt^2$ (Do not prove these.)



(i) Show that the particle reaches the highest point on its path when $t = \dfrac{V\sin\theta}{g}$. **2**

(ii) Show that the speed of projection is given by $V^2 = \dfrac{g}{2h}\left(4h^2 + c^2\right)$. **3**

(iii) Find the angle of projection, θ, in terms of h and c. **2**

End of paper