Centre Number

Student Number

CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2008
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Afternoon Session
Thursday, 14 August 2008

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1-7
- All questions are of equal value

3201 - 1

Total marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

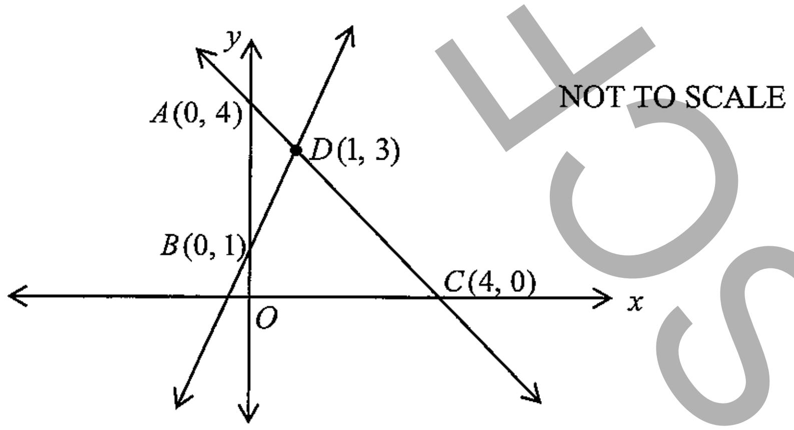
(a) Find the exact value of $\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$. **2**

(b) (i) Sketch the graph of $y = |2 - x|$. **1**

(ii) Using this graph, or otherwise, find the solution to $|2 - x| < x$. **2**

(c) Find the value of k if $x + 2$ is a factor of $P(x) = x^2 + kx + 6$. **2**

(d) $A(0, 4)$, $B(0, 1)$, $C(4, 0)$ and $D(1, 3)$ are points in the plane where D is the point of intersection of the two lines shown. Find, correct to the nearest minute, the size of the acute angle, $\angle BDC$, between the two lines. **3**



(e) Jasi was trying to find the solution to the inequality $\frac{3}{x+1} < 2$. **2**

He stated that the solution is all values of x greater than $\frac{1}{2}$.

Solve the inequality $\frac{3}{x+1} < 2$ to determine if Jasi's solution is correct.

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) If α, β and γ are the roots of $2x^3 - 5x^2 + 3x - 5 = 0$ find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. **2**

(b) Let $f(x) = \dfrac{2x}{\sqrt{1-x^2}}$.

(i) For what values of x is $f(x)$ undefined? **1**

(ii) Find $\displaystyle\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}}\,dx$, using the substitution $x = \sin u$. **3**

(c) (i) Find the derivative of $\sin^{-1}x + \cos^{-1}x$. **1**

(ii) Explain why $\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$. **2**

(d) How many different arrangements can be made from the letters of the word EXERCISE if:

(i) there are no restrictions? **1**

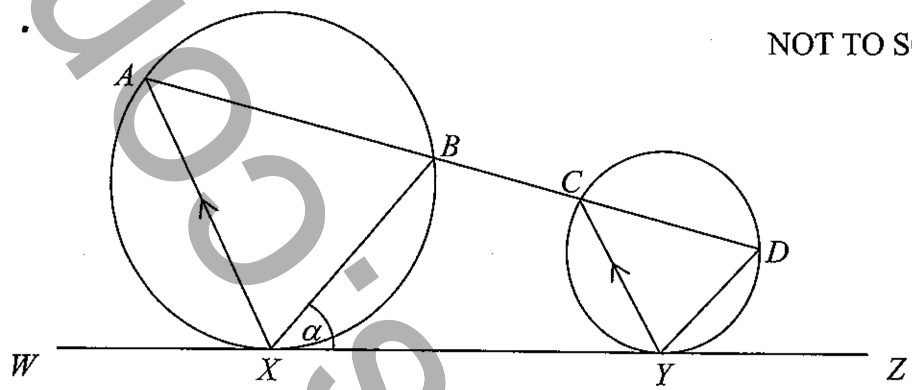
(ii) the letters C and R are at the ends? **2**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) In the diagram below, WZ is a common tangent to the two circles and AX is parallel to CY. AD is a straight line through B and C on the circles as shown. Let $\angle BXY = \alpha$.

NOT TO SCALE



Copy or trace this diagram into your writing booklet.

(i) Explain why BX is parallel to DY. **3**

(ii) Show that $BCYX$ is a cyclic quadrilateral. **1**

(b) If A and B are both reflex angles, and given $\cos A = \frac{3}{5}$ and $\tan B = \frac{12}{5}$, find the exact value of $\sin(A - B)$. **3**

(c) In the expansion of $(1 - kx)^9$ the coefficient of x^6 is half that of the coefficient of x^5. Find the value of the constant k. **3**

(d) Taking $x = 2$ as the first approximation, use one application of Newton's method to obtain a closer approximation to the solution to $x = \sqrt[3]{9}$. **2**

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Prove by mathematical induction that $\sum_{r=1}^{n} r \times r! = (n+1)! - 1$. **3**

(b) The acceleration of a particle P, moving in a straight line, is given by $\ddot{x} = 2x - 3$ where x metres is the displacement from the origin O. Initially the particle is at O and its velocity v is 2 metres per second.

(i) Show that the velocity v of the particle is $v^2 = 2x^2 - 6x + 4$. **2**

(ii) Calculate the velocity and acceleration of P at $x = 1$ and briefly describe the motion of P after it moves from $x = 1$. **2**

(c) The rate of change of the number of bees infected by a disease is given by the equation $\frac{dN}{dt} = N(200 - N)$, where N is the number of infected bees in the hive at time t years. There are 200 bees in the hive.

(i) If k is a constant, show that $N = \frac{200}{1 + ke^{-200t}}$ satisfies the above equation. **2**

(ii) If at time $t = 0$ one bee was infected, after how many days will half the colony be infected? **2**

(iii) Show that eventually all the bees will be infected. **1**

Marks

Question 5 (12 marks) Use a SEPARATE writing booklet

(a) Let $P(x) = -2x^3 + px^2 - qx + 5$.

(i) Show that if $P(x)$ is to have any stationary points, then $p^2 - 6q \geq 0$. **2**

(ii) Discuss the situation when $p^2 - 6q = 0$. **1**

(b) A camera, one kilometre away in the horizontal direction from where the Space Shuttle is being launched, is tracking the ascent of the Shuttle. Assume the Shuttle ascends vertically. **3**

Thirty seconds after the launch the Shuttle reaches a height, h, of 3240 metres and it is travelling at a speed of 230 metres per second.

The angle θ is the angle of elevation of the camera as it tracks the Shuttle. At what rate is θ increasing 30 seconds after the Shuttle is launched?

(c) Diana loves to play basketball. From the free throw line she makes 2 out of every 5 baskets that she throws. For every basket that she makes she scores one point.

(i) In her game last week she had 6 free throws. **1**
What is the probability that she scored 2 points?

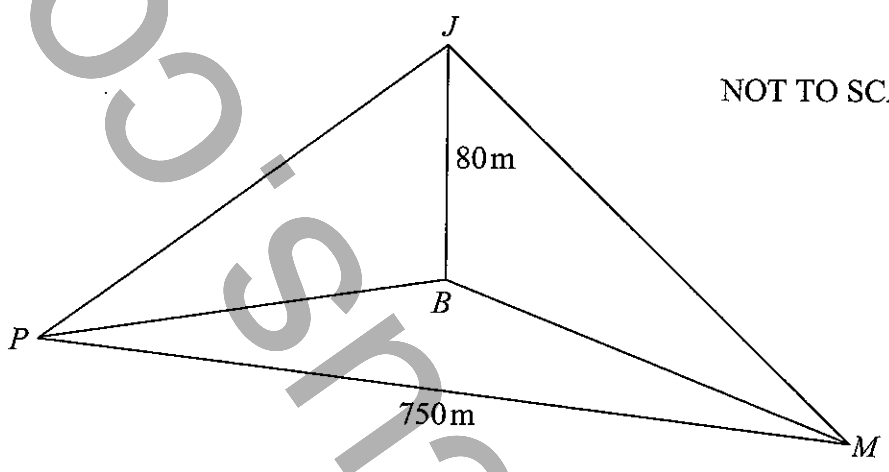
(ii) How many free throws would she need in one game so that the probability that she scores at least one point is 0.9978? **2**

Question 5 continues on page 7

Marks

(d) Janus, J, is on top of an 80 metre cliff, watching the Sydney to Hobart yacht race. **3**

From the base of the cliff, B, directly below Janus, *Poseidon*, P, is on a bearing of 202° and *Majorca*, M, is on a bearing of 140°. *Majorca* is 750 m from *Poseidon* on a bearing of 110°.



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Copy or trace this diagram into your writing booklet.

Find the angle of depression of *Poseidon*, P, from Janus, J.

End of Question 5

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet

(a) Consider the function given by $f(x)=\frac{e^x}{x-1}$.

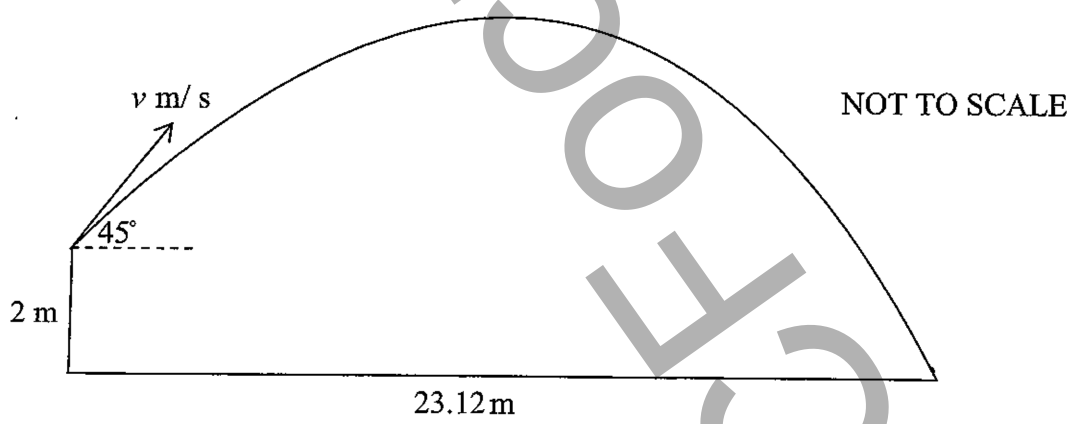
(i) Determine all vertical and horizontal asymptotes of the graph of $y=f(x)$. **2**

(ii) Find any stationary point(s) and sketch the graph of $y=f(x)$ including any intercepts with the coordinate axes. **3**

(iii) State the largest positive domain for which $f(x)$ has an inverse. **1**

(b) The world record for men's shot-put is 23.12 metres.

You may assume that the shot-put is projected at an initial velocity of v m/s from a height of 2 metres at an angle of projection of 45°, there is no air resistance and that the acceleration due to gravity is $10\,\text{m/s}^2$.



(i) Use integration to show that the equations of motion are **2**

$x=\frac{vt}{\sqrt{2}}$ and $y=-5t^2+\frac{vt}{\sqrt{2}}+2$.

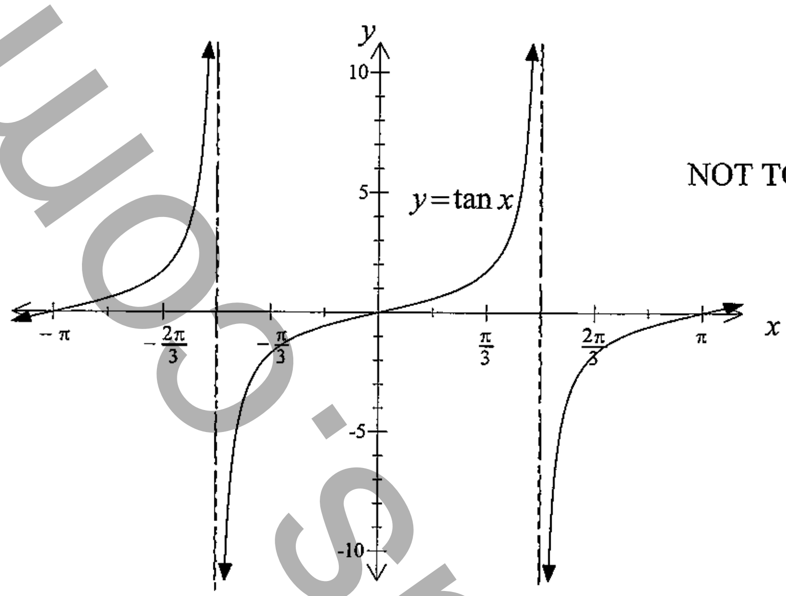
(ii) Find the minimum velocity v m/s at which the shot-put must be projected to achieve the world record distance. **2**

(iii) What is the maximum height that the shot-put reaches in its path if it is projected with this velocity? **2**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet

(a) The graphs shown are of $y = \tan x$ and $y = \sec x$ respectively.



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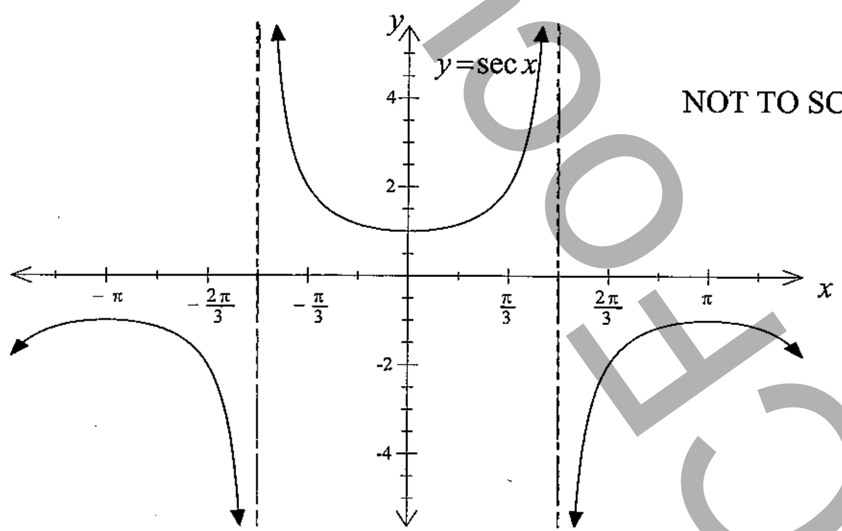

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(i) Prove that $\sec\theta - \tan\theta = \dfrac{1}{\sec\theta + \tan\theta}$. **2**

(ii) Explain why $0 < \sec\theta - \tan\theta \le 1$ for $0 \le \theta < \dfrac{\pi}{2}$. **2**

(iii) Solve the equation $\sec\theta - \tan\theta = \dfrac{1}{2}$ for $0 \le \theta < \dfrac{\pi}{2}$. **3**

Question 7 continues on page 10

Marks

Question 7 (continued)

(b) (i) From a point $A(p, q)$ perpendiculars AP and AQ are drawn to meet the x and y axes at $P(p, 0)$ and $Q(0, q)$ respectively. **1**

Find the equation of PQ.

(ii) Show that the condition for the line PQ to be a tangent to the parabola $x^2 = 4ay$ is $aq + p^2 = 0$. **3**

(iii) If the points $P(p, 0)$ and $Q(0, q)$ move on the x and y axes respectively such that PQ is a tangent to the parabola $x^2 = 4ay$ then the point $A(p, q)$ traces out a curve as P and Q move. **1**

Find the locus of A.

End of paper

Examiners

Carolyn Gavel (convenor)	Kambala, Rose Bay
Cynthia Athayde	St John Bosco College, Engadine
Joe Grabowski	Freeman Catholic College, Bonnyrigg
Anne Hastings	Kambala, Rose Bay
Br Domenic Xuereb fsp	Patrician Brothers' College, Fairfield

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2} \right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$