2007 TRIAL – EXT 1

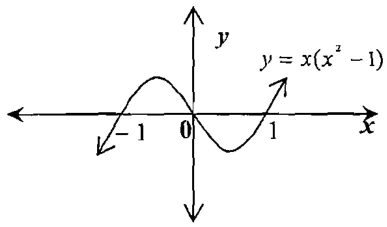
Marking Guidelines: Mathematics Extension I – Solutions

Question 1 (a)(b)(c)(d)(i)

Criteria	Marks
(a) One mark for the diagram and one for the solution.	2
(b)(i) One mark for the answer. **(ii)** One mark for the diagram (or explanation) and one mark for the answer.	3
(c) One mark for derivation.	1
(d) One mark for each step.	2
(e) One mark for finding the domain and range and one mark for the sketch.	2
(f) One mark for showing it's quarter of the area of circle and one for the answer.	2

Answers:

1(a)



From the graph, the solutions to

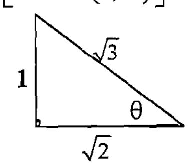
$x(x^2-1)>0$ are $-1<x<0$ and $x>1$

1(b)(i)

$\sin\frac{5\pi}{4}=\sin\left(\pi+\frac{\pi}{4}\right)$

$=-\sin\frac{\pi}{4}=-\frac{1}{\sqrt{2}}$

(ii) $\sin\left[2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]=\sin 2\theta$ from diagram



Now $\sin 2\theta=2\sin\theta\cos\theta$

$=2\times\frac{1}{\sqrt{3}}\times\frac{\sqrt{2}}{\sqrt{3}}=\frac{2\sqrt{2}}{3}$

$\therefore \sin\left[2\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]=\frac{2\sqrt{2}}{3}$

(c) $x=\sin\theta+\cos\theta$; $y=\sin\theta-\cos\theta$

$x^2=(\sin\theta+\cos\theta)^2$

$=\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta$

$=1+\sin 2\theta$

$y^2=\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta$

$=1-\sin 2\theta$

1(c) continued $\therefore\ x^2+y^2=2$

(d) $\lim_{x\to 0}\frac{3x}{\sin 2x}=\lim_{x\to 0}\left(\frac{3}{2}\times\frac{2x}{\sin 2x}\right)$

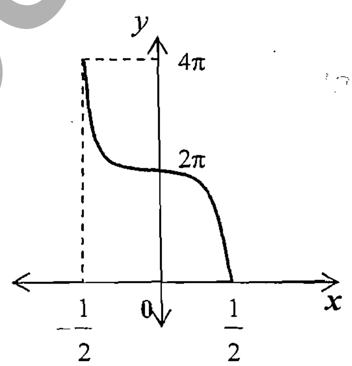
$=\frac{3}{2}\lim_{x\to 0}\left(\frac{2x}{\sin 2x}\right)=\frac{3}{2}$

as $\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)=1$

(e) $y=4\cos^{-1}2x$

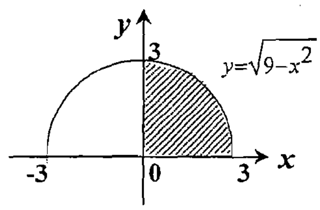
Domain: $-1\le 2x\le 1$ i.e. $-\frac{1}{2}\le x\le\frac{1}{2}$

Range: $0\le y\le 4\pi$



(g) $\int_0^3\sqrt{(9-x^2)}\,dx=$ shaded area

= quarter of circle of radius 3 units



$=\frac{9}{4}\pi$ unit^2

Question2

Criteria	Marks
(a) **(i)**One mark for tan(A+B).**(ii)**One mark each for deriving the quadratic equation and the solution.	3
(b) One mark for writing expression for the perpendicular distance and one for the coordinates.	2
(c) One mark for T_8 and one for the coefficient of a^5b^7.	2
(d) One mark each for x and y coordinate	2
(e) One mark for the table, one of writing Simpson's rule and one for application.	3

Answers

(a) (i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$\tan(A+B) = \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$

Dividing by $\cos A\cos B$ throughout we get,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

(ii) Given $\tan^{-1}x + \tan^{-1}2x = \frac{\pi}{4}$

$$\therefore \tan\left(\tan^{-1}x + \tan^{-1}2x\right) = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{\tan\left(\tan^{-1}x\right) + \tan\left(\tan^{-1}2x\right)}{1 - \tan\left(\tan^{-1}x\right)\times\tan\left(\tan^{-1}2x\right)} = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{x+2x}{1-2x^2} = 1 \Rightarrow 2x^2 + 3x - 1 = 0$$

$$\Rightarrow x = \frac{-3\pm\sqrt{9+8}}{4} = 0\cdot 28$$ (2 dec.pl.) taking the positive value as required.

(b) Let $(x, 2x)$ be the point on $y = 2x$

The distance of this point from $x + y - 4 = 0$ is given by

$\frac{x+2x-4}{\sqrt{2}}$ and this is given equal to $\pm\sqrt{2}$

$\therefore x + 2x - 4 = \pm 2 \Rightarrow 3x = 4 \pm 2$

or $x = 6/3$ or $2/3$ $\therefore x = 2$ and $\frac{2}{3}$

$\therefore$ the points are $(2,4)$ and $\left(\frac{2}{3}, \frac{4}{3}\right)$

Note: A point and its image on a line will be of equal length from the line but opposite in sign and hence the sign $\pm$.

(c) For the expansion $(2a-b)^{12}$,

$T_{r+1} = {}^{12}c_r(2a)^{12-r}(-b)^r$

The term a^5b^7 occurs when $r = 7$

Hence $T_8 = {}^{12}c_7(2a)^5(-b)^7$

$= \frac{12!}{5!7!}(2a)^5(-b)^7 = 792\times 32a^5\times -b^7$

$= -25344a^5b^7$

Coeff. of a^5b^7 is -25344

(d)

$x = \frac{-2(-4)+3(3)}{-2+3}$, $y = \frac{-2(2)+3(5)}{-2+3}$

$= \frac{8+9}{1}$ $\qquad = \frac{-4+15}{1}$

$= 17$ $\qquad = 11$ ie $(17, 11)$

(e)

x	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$
cos x	0	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{\sqrt{2}}$	0
	(y_0)	(y_1)	(y_2)	(y_3)	(y_4)

Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\{(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\}$$

where h = width of interval = $\pi/4$

$$\therefore \int_a^b \cos x\,dx \approx \frac{\pi}{12}\left\{0 + 4\times\frac{2}{\sqrt{2}} + 2\right\}$$

$$\approx \frac{\pi}{12}\left\{\frac{8}{\sqrt{2}} + 2\right\} \approx 2\cdot 00$$ (2dec.pl.)

Question 3 & 4(a)

Criteria	Marks
(a) One mark for the explanation and one for the answer.	2
(b)(i) One mark for showing it's SMH **(ii)** one mark each for α and a **(iii)** one mark for the position	4
(c)(i) &(ii) One mark for each answer **(iii)** One mark for showing blue marbles as a group = 4! and one for 4!4!	4
(d) One mark for writing the equation and one for finding the value of a	2
4(a) One mark for the solution.	1

Answers

3(a) Out of the 11 letters of MATHEMATICS, three letters – A, T and M are repeated.
Hence total no. of words with 11 letter

$$= \frac{11!}{2!2!2!} = 4989600$$

(b) (i) $x = a\cos(4t+\alpha) \quad \Rightarrow \dot{x} = -4a\sin(4t+\alpha)$

$\Rightarrow \ddot{x} = -16a\cos(4t+\alpha)$

$= -16 \times a\cos(4t+\alpha)$

$= -16x$

As the particle's motion can be described in the form $\ddot{x} = -n^2 x$, where $n = 4$, it is undergoing simple harmonic motion

(ii) $x = a\cos(4t+\alpha)$

when $t = 0$, $x = 0$, $\therefore$ $0 = a\cos(\alpha)$

i.e. $\cos\alpha = 0$ $\therefore$ $\alpha = \pi/2$

$\therefore\ x = a\cos\left(4t + \frac{\pi}{2}\right)$

and $v = \dot{x} = -4a\sin\left(4t + \frac{\pi}{2}\right)$

when $t = 0$, $v = -6$, $\therefore$ $-6 = -4a\sin\left(\frac{\pi}{2}\right)$

i.e. $4a = 6$ or $a = 3/2$ m

(iii) $x = \frac{3}{2}\cos\left(4t + \frac{\pi}{2}\right)$

when $t = 4$; $x = \frac{3}{2}\cos\left(16 + \frac{\pi}{2}\right) = 0{\cdot}43$m (2dec.pl)

the particle is about <u>0·43m to the right of origin</u>.

(c) (i) $7! = 5040$ arrangements

(ii) 7!– number of arrangements in which blue and white marbles alternate $= 7! - 3!4!$

$= 5040 - 144$

$= 4896$

(iii) blue marbles arranged as a group $= 4!$

blue marbles as a group and white marbles $= 4!$

$\therefore 4!4! = 576$

(d) Since it is a cubic with -1 as a repeated root and 2 as a root it can be written as

$y = a(x+1)^2(x-2)$

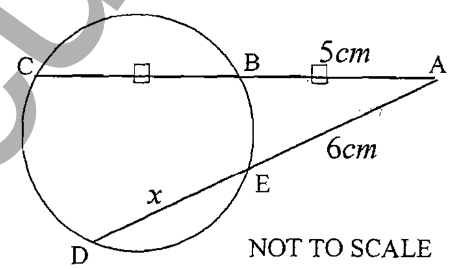
It passes through $(0, -4)$,

$-4 = a(0+1)^2(0-2) \quad \Rightarrow -4 = -2a$

$\therefore$ $a = 2$ and the equation of the curve is

$\underline{y = 2(x+1)^2(x-2)}$

4(a)



NOT TO SCALE

Since the chords of the circle, DE and CB intersect at A,

DA×EA=CA×BA

$\Rightarrow (6+x) \times 6 = 10 \times 5$

$(6+x) = 8\frac{1}{3}$ or $\underline{x = 2\frac{1}{3}}$

Question 4(b) –

Criteria	Marks
(b)(i) One mark for showing that the root lies between the two values indicated. **(ii)** One mark for the value of $f'(x)$ and one for applying Newton's method.	3
(c) One mark for finding the product of roots and one mark for finding the value of k.	2
(d) One mark for the tangent, one mark for $\angle$ APX, one for $\angle$ QPX and one for showing that AP bisects $\angle$BPQ.	4
(e) One mark for subtracting 1 and one for the solution	2

Answers

4(b) Let $f(x) = x + \sin x - \frac{\pi}{3}$

then $f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \frac{1}{2} - \frac{\pi}{3} \approx -0 \cdot 0236$

and $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} + \frac{1}{\sqrt{2}} - \frac{\pi}{3} \approx 0 \cdot 445$

Thus over the interval $\frac{\pi}{6} \le x \le \frac{\pi}{4}$, $f(x)$ changes from – ive to + ive. Hence it has at least one root between $\frac{\pi}{6}$ and $\frac{\pi}{4}$.

Let $x_1 = \frac{\pi}{6}$ then $f'(x) = 1 + \cos x$

and $f'\left(\frac{\pi}{6}\right) = 1 + \cos\frac{\pi}{6} = 1 + \frac{\sqrt{3}}{2}$

By Newton's law, $x_2 = \frac{\pi}{6} - \frac{f\left(\frac{\pi}{6}\right)}{f'\left(\frac{\pi}{6}\right)}$

$$= \frac{\pi}{6} - \frac{-0 \cdot 0236}{1 + \frac{\sqrt{3}}{2}} \approx 0.54\,(2\,\text{dp})$$

(c) Let α, 1/α and β be the roots.

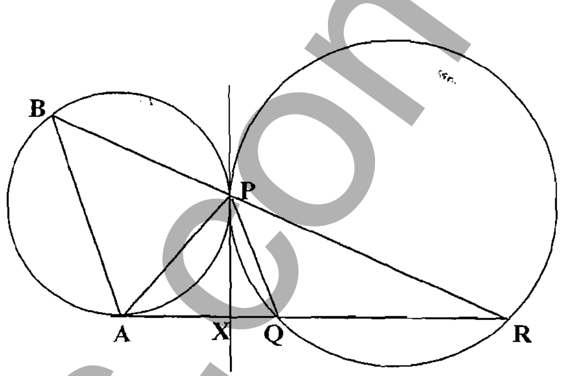
Product of roots = $\alpha \cdot 1/\alpha \cdot \beta = \frac{-4}{2} = -2$

i.e. $\beta = -2$

Since – 2 is a root, $2(-2)^3 - (-2)^2 + k(-2) + 4 = 0$

$\Rightarrow -2k = 16$ or $\underline{k = -8}$

(d) Construction : Draw through P a common tangent to the two circles to meet AR in X.



Proof: $\angle$XPA = $\angle$ABP (alt. seg) $= x^\circ$ say

$\angle$XPQ= $\angle$PRQ (alt.seg) $= y^\circ$ say

$\angle$APQ= $\angle$XPA+ $\angle$XPQ= $x^\circ + y^\circ$

Also $\angle$XAP=$\angle$ABP (alt.seg)

$\therefore$ $\angle$XAP=$\angle$XPA $= x^\circ$

Now in Δ APR, Ext. $\angle$APB = $\angle$PAR + $\angle$PRA $= x^\circ + y^\circ$

$\therefore$ $\angle$APQ = $\angle$APB

(e) $\frac{2x-3}{x} - 1 \le 0 \quad \Rightarrow \frac{2x-3-x}{x} \le 0$

$\Rightarrow \frac{x-3}{x} \le 0$

The critical points are 0 and 3.

When $x < 0$, $\frac{x-3}{x} > 0$

When $0 < x \le 3$, $\frac{x-3}{x} \le 0$

When $x > 3$, $\frac{x-3}{x} > 0$

$\therefore$ The solution is $\underline{0 < x \le 3}$

Question 5

Criteria	Marks
(a) One mark for conversion into cos 6x and one mark for integration.	2
(b) (i) One mark writing it's not a one to one function. **(ii)** One mark for the sketch. **(iii)** One mark for stating domain **(iv)** One mark for interchange of variables for finding y.	4
(c) (i) One mark for showing that $0 < y < 1$. **(ii)** one mark for the differentiation and one for simplifying.	3
(iii) One mark for substitution for P, q one for y and one mark for t.	3

Answers

5 (a) $\cos 2(3x) = 1 - 2\sin^2(3x)$

$\therefore \sin^2(3x) = \dfrac{1-\cos 2(3x)}{2}$

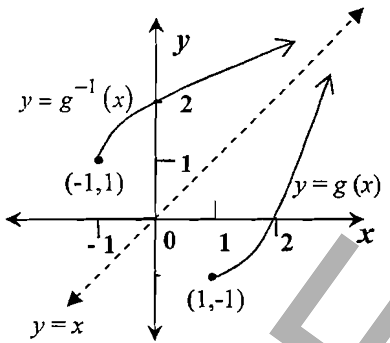
$\int \sin^2 3x\,dx = \int \dfrac{1-\cos 2(3x)}{2}\,dx$

$= \dfrac{1}{2}\int [1 - \cos 6x]\,dx$

$= \dfrac{1}{2}\left(x - \dfrac{\sin 6x}{6}\right) + c$

(b) (i) *f(x)* does not have an inverse because it is not a one to one function. That is, for at least one y-value there is more than one x-value.

(ii)



(iii) Domain of $y = g^{-1}(x)$ is $\underline{x \geq -1}$

(iv) Interchanging x and y , we get

$x = (y-1)^2 - 1$

or $y = \sqrt{x+1} + 1$ i.e $g^{-1}(x) = \sqrt{x+1} + 1$

(c) (i) $y = \dfrac{Pe^{qt}}{1+Pe^{qt}}$, $P > 0, q > 0$

Since $Pe^{qt} > 0$ for all t

then $1 + Pe^{qt} > Pe^{qt}$ $\therefore$ $\dfrac{Pe^{qt}}{1+Pe^{qt}} < 1$

or $y > 0$ also since $Pe^{qt} > 0$ **&** $1 + Pe^{qt} > 0$

$y = \dfrac{Pe^{qt}}{1+Pe^{qt}} > 0$ Hence $\underline{0 < y < 1}$

(ii) $\dfrac{dy}{dt} = \dfrac{Pe^{qt}\cdot q(1+Pe^{qt}) - Pe^{qt}(Pe^{qt}\cdot q)}{(1+Pe^{qt})^2}$

$= q\left[\dfrac{Pe^{qt}(1+Pe^{qt})}{(1+Pe^{qt})^2} - \dfrac{(Pe^{qt})^2}{(1+Pe^{qt})^2}\right]$

$= q\left[\dfrac{Pe^{qt}}{1+Pe^{qt}} - \left(\dfrac{Pe^{qt}}{1+Pe^{qt}}\right)^2\right]$

$= q(y - y^2) = q\,y(1-y)$

(iii)

Given P=0·01and q=0·7,to find t when y =0·5

Substituting these values in $y = \dfrac{Pe^{qt}}{1+Pe^{qt}} > 0$ we get

$0\cdot 5 = \dfrac{0\cdot 01e^{0\cdot 7t}}{1+0\cdot 01e^{0\cdot 7t}} \Rightarrow 0\cdot 5\times(1+0\cdot 01e^{0\cdot 7t}) = 0\cdot 01e^{0\cdot 7t}$

$\Rightarrow 0\cdot 5 = 0\cdot 0\,05e^{0\cdot 7t} \Rightarrow e^{0\cdot 7t} = 100$

$\therefore t = \dfrac{\ln 100}{0\cdot 7} \cong 6\cdot 58$ or $\underline{t = 7}$ hours to nearest hour

Question 6

Criteria	Marks
(a)(i) One mark for integrating to get v^2, one for finding v and one for expressing x in terms of t. **(ii)** One mark for finding the displacement of P.	4
(b)(i) One mark for the expression showing the volume and one for simplifying it **(ii)** One mark for finding an expression for h, one for finding $d\theta/dV$, one for finding the value of θ when $x = 0{\cdot}3$ and one mark for the carrying out the calculations.	6
(c) One mark for finding the derivative and one for the value of x for which $f(x)$ decreases.	2

Answers

6(a)(i) $\ddot{x} = 8x\left(x^2+1\right)$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 8x\left(x^2+1\right) \Rightarrow \frac{1}{2}v^2 = \frac{8x^4}{4} + \frac{8x^2}{2} + c_1$$

When $x = 0$; $v = -2$ $\therefore$ $\frac{4}{2} = 0 + 0 + c_1$ or $c_1 = 2$

$$\frac{1}{2}v^2 = 2x^4 + 4x^2 + 2 = 2\left(x^2+1\right)^2$$

$\therefore$ $v = \pm 2\left(x^2+1\right)$ or speed = $\underline{v = 2\left(x^2+1\right) cm/s}$

$$v = \frac{dx}{dt} = \pm 2\left(x^2+1\right) \Rightarrow \pm dt = \frac{dx}{2\left(1+x^2\right)}$$

$$\therefore \pm t = \frac{1}{2}\int \frac{dx}{1+x^2} = \frac{1}{2}\tan^{-1} x + c_2$$

when $t = 0$, $x = 0$ $\therefore$ $c_2 = 0$

$\therefore$ $\pm t = \frac{1}{2}\tan^{-1} x$ $\Rightarrow x = \tan(\pm 2t) = \pm\tan 2t$

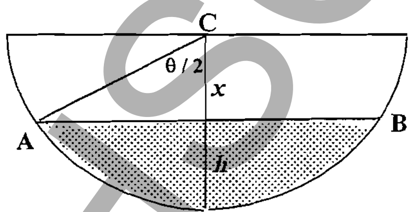
(ii) When $t = \pi/8$, $x = \pm\tan \pi/4 = \pm 1$
The particle is 1 cm from O.

(b) (i) Volume of water(V)
= (area of sector ACB – Area of Δ ACB)$\times$ length

$$= \left(\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\right)\times 2$$

$$= \left(\frac{1}{8}\theta - \frac{1}{8}\sin\theta\right)\times 2 = \frac{1}{4}\left(\theta - \sin\theta\right)$$

(ii)



(b)(ii) continued.

From the diagram, $x = \frac{1}{2}\cos\frac{\theta}{2}$

$$h = \frac{1}{2} - x = \frac{1}{2} - \frac{1}{2}\cos\frac{\theta}{2}$$

$$\therefore \frac{dh}{d\theta} = \frac{1}{4}\sin\frac{\theta}{2}$$

$$V = \frac{1}{4}\left(\theta - \sin\theta\right) \Rightarrow \frac{dV}{d\theta} = \frac{1}{4}\left(1-\cos\theta\right)$$

and $\frac{d\theta}{dV} = \frac{4}{1-\cos\theta}$

To find dh/dt when $h = 0{\cdot}2$ m and $dV/dt = 0{\cdot}1$

Now $\frac{dh}{dt} = \frac{dh}{d\theta}\times\frac{d\theta}{dV}\times\frac{dV}{dt}$

$$= \frac{1}{4}\sin\frac{\theta}{2}\times\frac{4}{1-\cos\theta}\times 0{\cdot}1$$

When $h = 0{\cdot}2$, $x = 0{\cdot}5 - 0{\cdot}2 = 0{\cdot}3$

substituting $x = 0{\cdot}3$ in $x = \frac{1}{2}\cos\frac{\theta}{2}$ we get

$$\cos\frac{\theta}{2} = 0{\cdot}6 \Rightarrow \frac{\theta}{2} = \cos^{-1} 0{\cdot}6 \therefore \theta = 1{\cdot}855$$

$$\frac{dh}{dt} = \frac{1}{4}\sin\frac{1{\cdot}855}{2}\times\frac{4}{1-\cos 1{\cdot}855}\times 0{\cdot}1$$

=0·0625 m / min (3sig.fig)

(c) $f(x) = xe^{-2x}$ Hence $f'(x) = e^{-2x} - 2xe^{-2x}$

i.e. $f'(x) = e^{-2x}\left(1-2x\right)$

$f(x)$ decreases when $f'(x) < 0$

i.e. $e^{-2x}\left(1-2x\right) < 0$

Now $e^{-2x} > 0$ for all values of x and

$1 - 2x < 0$ when $x > \frac{1}{2}$

$\therefore f(x)$ decreases for $\underline{x > \frac{1}{2}}$

Question 7

Criteria	Marks
(a)(i) One mark for showing no.of ways of selecting 3f and 1m **(ii)** One mark for showing no. ways with 3f. **(iii)** One mark for the explanation and answer. **(b)** One mark for substitution, one for integration.	5
(c)(i) One mark for deriving an expression for time for max. height and one mark for showing V sin α in terms of *g* and *h* **(ii)** One mark for the quadratic equation, one mark for its solution (*t*) and one for finding the horizontal distance	5
(d)(i) One mark for showing that it is the binomial expansion of $[x+(1-x)]^n$ that equals to 1 **(ii)** One mark for the substitution for *x* and simplification.	2

Answers

7(i) The number of ways of selecting the 2 females from *n* females is ${}^nc_2$ and the number of selecting 1 male for *n* males is ${}^nc_1$.

$\therefore$ No. of ways of selecting 1 male and 2 females =

${}^nc_1 \times {}^nc_2 = n\left({}^nc_2\right)$ since ${}^nc_1 = n$

(ii) No. of ways of selecting 3 females from *n* females is ${}^nc_3$

(iii) Total no. of ways of selecting a committee of 3 people from 2*n* people is ${}^{2n}c_3$.

This total can be divided into 4 categories:-
3 males, 0 females; 2 males,1 female;
1 male, 2 females; 0 males, 3 females

$\therefore$

$${}^{2n}c_3 = \left({}^nc_3 \times {}^nc_0\right)+\left({}^nc_2 \times {}^nc_1\right)+\left({}^nc_1 \times {}^nc_2\right)+\left({}^nc_0 \times {}^nc_3\right)$$
$$= 2\left({}^nc_3\right)+\left({}^nc_2 \times n\right)+\left(n \times {}^nc_2\right)$$
$$= 2\left({}^nc_3 + n\,{}^nc_2\right)$$

or $n({}^nc_2) + {}^nc_3 = \frac{1}{2}\left({}^{2n}c_3\right)$

(b)

$\int x\sqrt{x^2+9}\,dx$ let $u = x^2+9$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

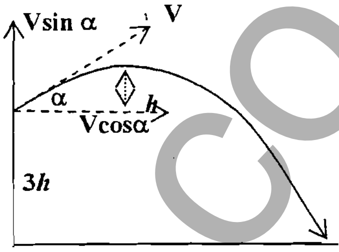
$$= \frac{1}{2}\int u^{\frac{1}{2}}du$$
$$= \frac{1}{2}\times\frac{2u^{\frac{3}{2}}}{3}+c$$
$$= \frac{(x^2+9)^{\frac{3}{2}}}{3}+c$$

(c) (i) Horizontal motion: $\ddot{x}=0$, Integrating we get

$\dot{x} = c_1$ when $t=0$, $\dot{x} = V\cos\alpha$ $\therefore c_1 = V\cos\alpha$

$\dot{x} = V\cos\alpha$

(c) (i) continued



Integrating again we get,
$x = Vt\cos\alpha + c_2$
when $t=0$, $x=0$ $\therefore c_2=0$
or $x = Vt\cos\alpha$
Vertical motion:
$\ddot{y}=-g$, Integrating we get,
$\dot{y}=-gt+c_3$
when $t=0$, $\dot{y}=V\sin\alpha$ and so $c_3 = V\sin\alpha$
$\therefore \dot{y}=-gt+V\sin\alpha$, Integrating again we get

$$y=-\frac{1}{2}gt^2+Vt\sin\alpha+c_3$$

when $t=0$, $y=3h$ $\therefore c_3=3h$

$$\therefore\ y=-\frac{1}{2}gt^2+Vt\sin\alpha+3h$$

Greatest height is reached when $\dot{y}=0$

$$-gt+V\sin\alpha=0 \text{ or } t=\frac{V\sin\alpha}{g}$$

When $t=\frac{V\sin\alpha}{g}$, $y=4h$

$$\therefore\ 4h=-\frac{g}{2}\left[\frac{V\sin\alpha}{g}\right]^2+V\left[\frac{V\sin\alpha}{g}\right]\sin\alpha+3h$$

$$h=-\frac{1}{2}\frac{V^2\sin^2\alpha}{g}+\frac{V^2\sin^2\alpha}{g}$$
$$=\frac{1}{2g}V^2\sin^2\alpha \Rightarrow V\sin\alpha=\sqrt{2gh}$$

7(c)(ii).

We substitute $y=0$, in $y=-\frac{1}{2}gt^2+Vt\sin\alpha+3h$ to get the time taken for the journey.

$$-\frac{1}{2}gt^2+Vt\sin\alpha+3h=0$$

$$\Rightarrow \quad gt^2-2Vt\sin\alpha-6h=0$$

$$\therefore \quad t=\frac{2\sqrt{2gh}\pm\sqrt{8gh+24gh}}{2g}$$

$$=\frac{2\sqrt{2gh}\pm 4\sqrt{2gh}}{2g}$$

or $\quad t=\dfrac{6\sqrt{2gh}}{2g}=3\sqrt{\dfrac{2h}{g}}$

Horizontal distance (or the time $t=3\sqrt{\frac{2h}{g}}$)

$$x=Vt\cos\alpha=V\sin\alpha\left(\frac{\cos\alpha}{\sin\alpha}\right)\times 3\sqrt{\frac{2h}{g}}$$

$$=\sqrt{2gh}\times\cot\alpha\times 3\sqrt{\frac{2h}{g}}=\underline{6h\cot\alpha}$$

(d)(i)

$$\left[x+(1-x)\right]^n={}^nc_0x^n+{}^nc_1x^{n-1}(1-x)+{}^nc_2x^{n-2}(1-x)^2+\cdots+{}^nc_n(1-x)^n$$

Now $\left[x+(1-x)\right]^n=\left[x+1-x\right]^n=1^n=1$

$$\therefore {}^nc_0x^n+{}^nc_1x^{n-1}(1-x)+{}^nc_2x^{n-2}(1-x)^2+{}^nc_2x^{n-2}(1-x)^2+\cdots+{}^nc_n(1-x)^n=1$$

(ii) Substituting $x=\frac{1}{2}$ in the above equation, we get

$${}^nc_0\left(\frac{1}{2}\right)^n+{}^nc_1\left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right)+{}^nc_2\left(\frac{1}{2}\right)^{n-2}\left(\frac{1}{2}\right)^2+\cdots+{}^nc_n\left(\frac{1}{2}\right)^n=1$$

$$\Rightarrow {}^nc_02^{-n}+{}^nc_12^{-n}+{}^nc_22^{-n}+\cdots+{}^nc_n2^{-n}=1$$

$$\Rightarrow {}^nc_0+{}^nc_1+{}^nc_2+\cdots+{}^nc_n=\underline{2^n}$$