SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2007

TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION

Mathematics   Extension 1

General Instructions

- Reading Time – 5 Minutes
- Working time – 2 Hours
- Write using black or blue pen. Pencil may be used for diagrams.
- Board-approved calculators maybe used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.
- Each Question is to be returned in a separate bundle.

Total Marks – 84

- Attempt Questions 1 – 7.
- All questions are of equal value.

Examiner: *A. Fuller*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

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Total marks - 84
Attempt Questions 1-7
All questions are of equal value

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a)   Evaluate $\lim\limits_{x \to 0} \dfrac{\sin 4x}{5x}$.   1

(b)   Calculate the acute angle (to the nearest minute) between the lines   2
        $2x + y = 4$ and $x - 3y = 6$.

(c)   (i)   Show that $x + 1$ is a factor of $x^3 - 4x^2 + x + 6$.   1

        (ii)   Hence, or otherwise factorise $x^3 - 4x^2 + x + 6$ fully.   2

(d)   The point $P(5,7)$ divides the interval joining the points $A(-1,1)$ and   2
        $B(3,5)$ externally in the ratio $k:1$. Find the value of k.

(e)   Find the horizontal asymptote of the function $y = \dfrac{3x^2 - 4x + 1}{2x^2 - 1}$.   1

(f)   Find a primitive of $\dfrac{1}{\sqrt{4 - x^2}}$.   1

(g)   Solve the equation $|x + 1|^2 - 4|x + 1| - 5 = 0$.   2

Question 2 (12 marks)

(a)   Let $f(x) = \dfrac{1}{2}\cos^{-1}\left(\dfrac{x}{3}\right)$.

         (i)   State the domain and range of the function $f(x)$.   **2**

         (ii)   Show that $y = f(x)$ is a decreasing function.   **2**

         (iii)   Find the equation of the tangent to the curve $y = f(x)$ at the   **2**
               point where $x = 0$.

(b)   Find the derivative of $y = \ln\left(\sin^3 x\right)$.   **2**

(c)   (i)   Write $\cos x - \sqrt{3}\sin x$ in the form $A\cos(x + \alpha)$, where $A > 0$ and   **2**
               $0 < \alpha < \dfrac{\pi}{2}$.

         (ii)   Hence, or otherwise, solve $\cos x - \sqrt{3}\sin x + 1 = 0$ for $0 \le x \le 2\pi$.   **2**

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Show that the equation $e^x - x - 2 = 0$ has a solution in the interval $1 < x < 2$. **1**

(ii) Taking an initial approximation of $x = 1.5$ use one application of Newton's method to approximate the solution, correct to three decimal places. **2**

(b) The normal at $P\left(2ap, ap^2\right)$ on the parabola $x^2 = 4ay$ cuts the y-axis at Q and is produced to a point R such that $PQ = QR$.

(i) Show that the equation of the normal at P is $x + py = 2ap + ap^3$. **2**

(ii) Find the coordinates of Q. **1**

(iii) Show that R has coordinates $\left(-2ap, ap^2 + 4a\right)$. **1**

(iv) Show that the locus of R is a parabola, and find its vertex. **3**

(c) If $\int_1^5 f(x)dx = 3$, find $\int_1^5 \left(2f(x) + 1\right)dx$. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a)   Using the substitution $u = e^x$, or otherwise, find $\int e^{\left(e^x + x\right)} dx$ 3

(b)   The velocity-time graph below shows the velocity of a lift as it travels
from the first floor to the twentieth floor of a tall building during the
T seconds of its motion.
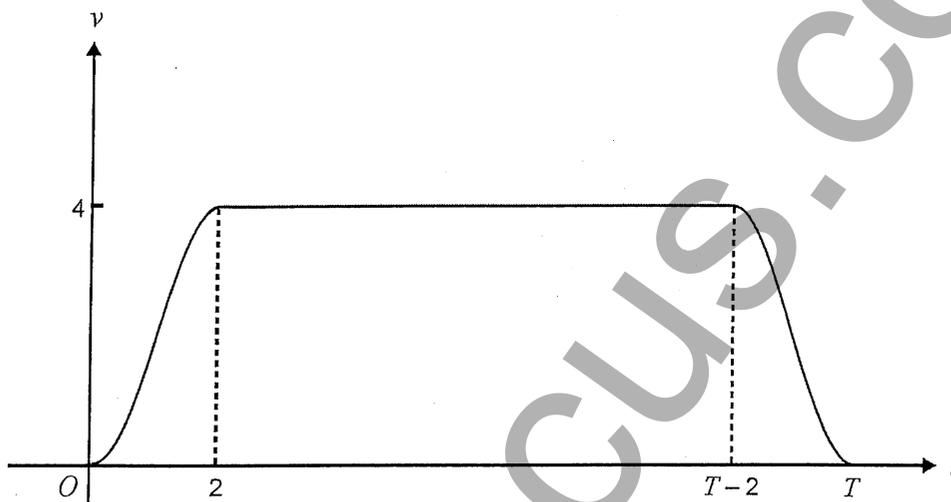
The velocity v m/s at time t s for $0 \le t \le 2$ is given by $v = t^2(3 - t)$. After the
First two seconds, the lift moves with a constant velocity of 4 m/s for a time,
and then decelerates to rest in the final two seconds.

The velocity-time graph is symmetrical about $t = \dfrac{1}{2}T$.

(i)   Express the acceleration in terms of t for the first two seconds of the   1
motion of the lift.

(ii)   Hence, find the maximum acceleration of the lift during the first two   2
seconds of its motion.

(iii)   Given that the total distance travelled by the lift during its journey is   2
41 metres, find the exact value of T.

(c)   A solid is formed by rotating about the y-axis the region bounded by the curve
        $y = \cos^{-1} x$, the x-axis and the y-axis.

        (i)   Show that the volume of the solid is given by $V = \pi \int_0^{\frac{\pi}{2}} \cos^2 y\, dy$.   **1**

        (ii)   Calculate the volume of this solid.   **3**

Question 5 (12 marks) Use a SEPARATE writing booklet.

(a)   Use mathematical induction to prove that $\displaystyle\sum_{r=1}^{n} r \times r! = (n+1)! - 1$.   **3**

(b)   In the expansion of $\left(2x + \dfrac{1}{x^2}\right)^{15}$, determine the coefficient of the term that   **3**

is independent of x.

(c)   The acceleration of a particle P is given by the equation $a = 8x\left(x^2 + 1\right)$,

where x is the displacement of P from the origin in metres after t seconds,

with movement being in a straight line.

Initially, the particle is projected from the origin with a velocity of

2 metres per second in the negative direction.

(i)   Show that the velocity of the particle can be expressed as   **2**

$v = 2\left(x^2 + 1\right)$.

(ii)   Hence, show that the equation describing the displacement   **2**

of the particle at time t is given by $x = \tan 2t$.

(iii)   Determine the velocity of the particle after $\dfrac{\pi}{8}$ seconds.   **2**

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a)
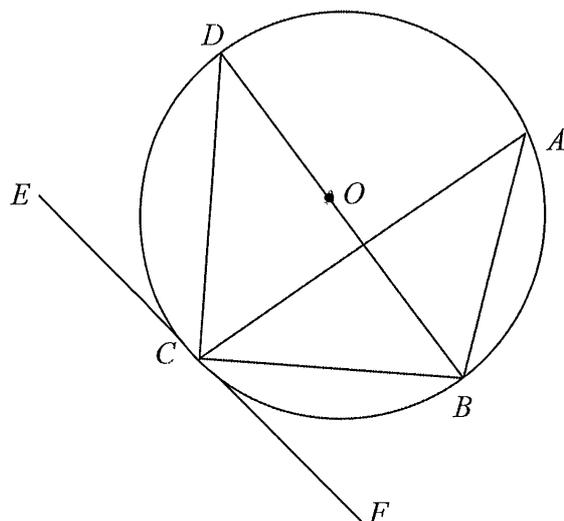
A, B, C and D are points on the circumference of a circle with centre O.   **3**

EF is a tangent to the circle at C and the angle ECD is $60°$.

Find the value of $\angle BAC$ giving reasons.

(b) (i) By considering the expansion of $(1+x)^n$ in ascending powers of x,   **1**

where n is a positive integer, and differentiating, show that

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \dots + n\binom{n}{n} = n(2^{n-1}).$$

(ii) Hence, find an expression for $2\binom{n}{1} + 3\binom{n}{2} + 4\binom{n}{3} + \dots + (n+1)\binom{n}{n}$.   **2**

(c) If $f(x+2) = x^2 + 2$, find $f(x)$.   **2**

(d) At a particular dinner, each rectangular table has nine seats, five facing the stage and four with their backs to the stage.

In how many ways can 9 people be seated at the table if

(i) John and Mary sit on the same side?   **2**

(ii) John and Mary sit on opposite sides?   **2**

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a)   A skier accelerates down a slope and then skis up a short jump (see diagram).
The skier leaves the jump at a speed of 12 m/s and at an angle of 60° to the
horizontal. The skier performs various gymnastic twists and lands on a straight
line section of the 45° down-slope T seconds after leaving the jump.
Let the origin O of a Cartesian coordinate system be at the point where the skier
leaves the jump. Displacements are measured in metres and time in seconds.
Let $g = 10ms^{-2}$ and neglect air resistance.
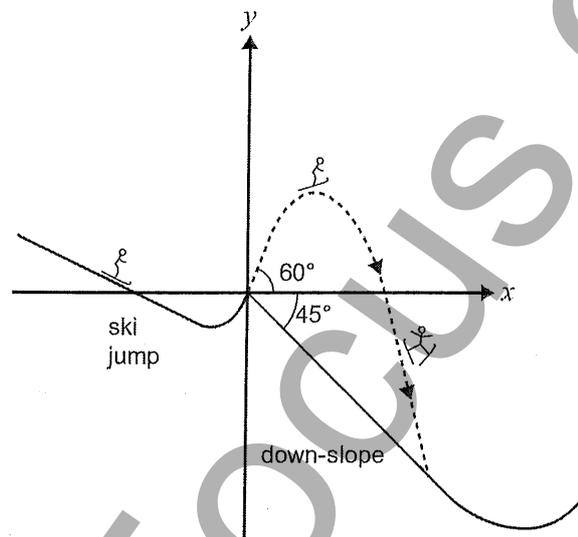
(i)   Derive the cartesian equation of the skiers flight as a function of y **3**
in terms of x.

(ii)   Show that $T = \dfrac{6}{5}\left(\sqrt{3} + 1\right)$.   **3**

(iii)   At what speed, in metres per second does the skier land on the   **2**
down-slope? Give your answer correct to one decimal place.

(b)
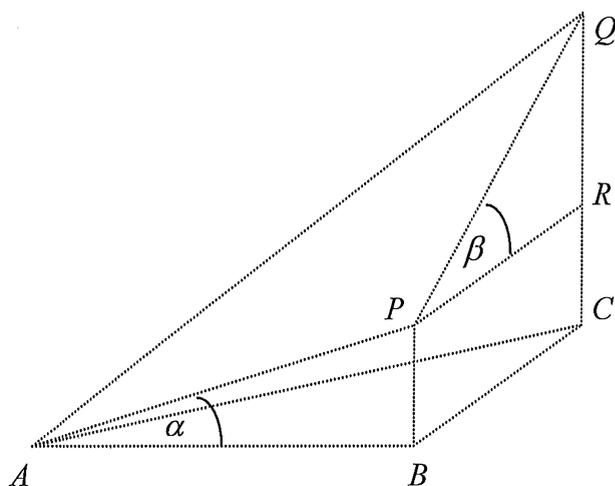
ABC is a horizontal, right-angled, isosceles triangle where $AB = BC$
and $\angle ABC = 90°$. P is vertically above B; Q is vertically above C.

The angle of elevation of P from A, and Q from P are α and β respectively.

(i) If the angle of elevation of Q from A is θ, prove that **2**

$$\tan\theta = \frac{\tan\alpha + \tan\beta}{\sqrt{2}}.$$

(ii) If $\angle APQ = \phi$, prove that $\cos\phi = -\sin\alpha\sin\beta$. **2**

End of Paper

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \ n \neq -1; \ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \ x > 0$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \ x > 0$