Question 1.   Yearly 2006.

(a) $(3\sqrt{2}-2\sqrt{3})(2\sqrt{2}+5\sqrt{3})$

$= 12 - 4\sqrt{6} + 15\sqrt{6} - 30$

$= 11\sqrt{6} - 18$

$= x + \sqrt{y}$

$\therefore x = 18$

$\therefore y = 121 \times 6 = 726$

(b) $5^{a+b} = 125 = 5^3$

$7^{a-b} = 1 = 7^0$

$\therefore a+b = 3$

$a-b = 0$

$2a = 3$

$a = \frac{3}{2}$

$\therefore b = \frac{3}{2}$

(c) $\frac{4x-1}{x-1} \geq 3$ $x \neq 1$

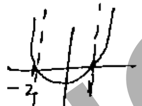
$(x-1)(4x-1) \geq 3(x-1)^2$

$4x^2 - 5x + 1 \geq 3(x^2 - 2x + 1)$

$4x^2 - 5x + 1 \geq 3x^2 - 6x + 3$

$x^2 + x - 2 \geq 0$

$(x+2)(x-1) \geq 0$



$\therefore$ solution is $x > 1$ and $x \leq -2$

(d) $(1.017)^n > 2$

try $n = 6$, $(1.017)^6 = 1.1064$

$n = 20$ $(1.017)^{20} = 1.4\ldots$

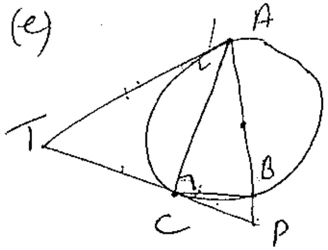
$n = 40$ $(1.017)^{40} = 1.962\ldots$

$n = 41$ $(1.017)^{41} = 1.995.$

$n = 42$ $(1.017)^{42} = 2.02$

$\therefore n = 42$

(e)



(i) $\angle BCP = \angle CAB$ (equal angles in alternate segment)

$\angle TAB = 90^\circ$ (tangent meets radius at 90°)

$\therefore \angle CAT = \angle TAB - \angle CAB$

$= 90^\circ - \angle BCP$

(ii) $\angle ACT = \angle CAT$ (ΔATC is isosceles because tangents from an external point are equal)

$\angle ATC = 180^\circ - 2\angle CAT$ (sum of $\angle$'s in Δ)

$= 180^\circ - 2(90^\circ - \angle BCP)$ from above

$= 2\angle BCP$

Question 2

a) $\frac{1+\cot^2 225^\circ}{\sec 300^\circ \operatorname{cosec} 750^\circ}$

$= \frac{1 + \frac{1}{\tan^2 225^\circ}}{\frac{1}{\cos 300^\circ} \cdot \frac{1}{\sin 750^\circ}}$

$= \frac{1 + \frac{1}{1^2}}{\frac{1}{\frac{1}{2}} \times \frac{1}{\frac{1}{2}}}$

$= \frac{1+1}{2 \times 2}$

$= \frac{2}{4}$

$= \frac{1}{2}$

(b) $x^{a^2+ab-ac} \cdot x^{-ba-b^2+bc} \cdot x^{b^2+bc-ab} \cdot x^{-cb-c^2+ac} \cdot x^{c^2+ac-bc} \cdot x^{-ac-a^2+ab}$

$= x^{a^2+ab-ac-ba-b^2+bc+b^2+bc-ab-cb-c^2+ac+c^2+ac-bc-ac-a^2+ab}$

$= x^0$

$= 1$

(c) $\log_{x-2}(2x-1) = 2$

$(x-2)^2 = 2x-1$

$x^2 - 4x + 4 = 2x - 1$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$\therefore x = 5,\ x = 1$

check $x=1$: $\log_{-1}(2x-1) = 2$ (no negative bases)

check $x = 5$, $\log_3(9) = 2$.

$\therefore x = 5$

(d) $2x + 3y - 6 = 0$

x	0	3
y	2	0

Test $(0,0)$ for $2x + 3y > 6$

$0 + 0 > 6$ NO

shade other side.

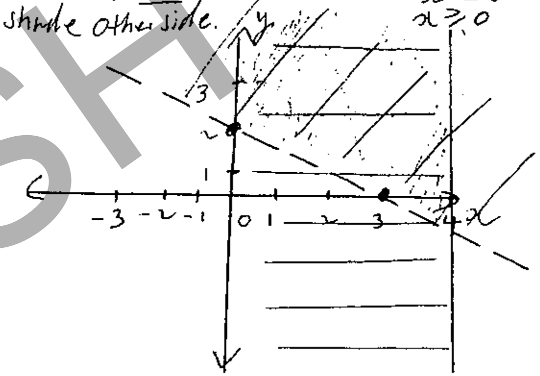
$|x-2| \leq 2$

$x - 2 \leq 2$ or $-(x-2) \leq 2$

i.e. $x \leq 4$ or $-x + 2 \leq 2$

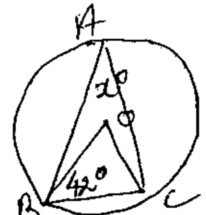
$-x \leq 0$

$x \geq 0$



//// is the region where $2x + 3y > 6$ and $|x-2| \leq 2$

(e)



$\angle BOC = 180^\circ - 2 \times 42^\circ$ (isosceles triangle due to equal radii and sum of $\angle$'s in a triangle)

$= 96^\circ$

$\angle BAC = x = \frac{1}{2} \times 96^\circ$ (angle at centre is twice angle at circumference standing on same arc)

$= 48^\circ$

(a) $f(x)=\sqrt{9-x^2}$

Domain is $-3 \le x \le 3$

Range is $0 \le f(x) \le 3$

(b) $f(u)=\frac{1}{u}$

$f(u+1)-f(u)=\frac{1}{u+1}-\frac{1}{u}$

$=\frac{u-(u+1)}{u(u+1)}$

$=\frac{-1}{u(u+1)}$

(c) (i) $\lim_{x\to 3}\frac{x^2-9}{x-3}=\lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)}$

$=\lim_{x\to 3}(x+3)$

$=6$

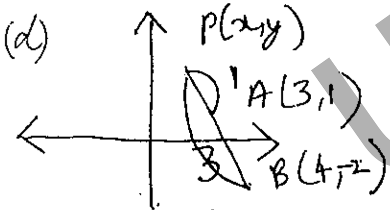
(ii) $\lim_{x\to\infty}\frac{12x^2-9x+2}{7x^2+4x-9}$

$=\lim_{x\to\infty}\frac{\frac{12x^2}{x^2}-\frac{9x}{x^2}+\frac{2}{x^2}}{\frac{7x^2}{x^2}+\frac{4x}{x^2}-\frac{9}{x^2}}$

$=\lim_{x\to\infty}\frac{12-\frac{9}{x}+\frac{2}{x^2}}{7+\frac{4}{x}-\frac{9}{x^2}}$

$=\frac{12}{7}$

(d)



$m:n=1:-3$

$(x_1, y_1)=(3,1)$

$(x_2, y_2)=(4,-2)$

$\therefore x=\frac{mx_2+nx_1}{m+n}$

$=\frac{1\times 4 \; -3\times 3}{-1 \; -3}$

$=\frac{-5}{-2}$

$=+2\frac{1}{2}$

$y=\frac{my_2+ny_1}{m+n}$

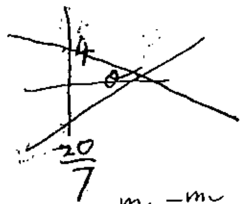
$=\frac{1\times -2 \; +-3\times 1}{1-3}$

$=\frac{-2 \; -3}{-2}$

$=2\frac{1}{2}$

$\therefore P$ is $(2\frac{1}{2}, 2\frac{1}{2})$

(e) $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A.\tan B}$



$y=-x+4$, $m_1=-1$

$y=\frac{x-20}{7}$ $m_2=\frac{1}{7}$

$\tan\theta=\frac{m_1-m_2}{1+m_1m_2}$

$=\left|\frac{-1-\frac{1}{7}}{1+-1\times\frac{1}{7}}\right|$

$=\left|\frac{-\frac{8}{7}}{\frac{6}{7}}\right|$

$=\left|-\frac{8}{6}\right|$

$\theta=\angle APB=53^0 8'$

Question 4.

(a) (i) $y=x^2-2x-\frac{1}{x^2}$

$=x^2-2x-x^{-2}$

$\frac{dy}{dx}=2x-2+2x^{-3}$

$=2x-2+\frac{2}{x^3}$

(ii) $y=\frac{5}{\sqrt{x}}$

$=5\times x^{-\frac{1}{2}}$

$\frac{dy}{dx}=5\times -\frac{1}{2}x^{-\frac{3}{2}}$

$=\frac{-5}{2\sqrt{x^3}}$

(iii) $y=\sqrt{2x+9}$

$=(2x+9)^{\frac{1}{2}}$

$\frac{dy}{dx}=\frac{1}{2}(2x+9)^{-\frac{1}{2}}.2$

$=\frac{1}{\sqrt{2x+9}}$ $\left[\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}\right]$

(iv) $y=(2x+3)(4x^2-7)^3$

$u=2x+3$

$v=(4x^2-7)^3$

$\frac{dy}{dx}=(2x+3).3(4x^2-7)^2.8x+(4x^2-7)^3.2$

$=2(4x^2-7)^2[12x(2x+3)+(4x^2-7)]$

$=2(4x^2-7)^2[24x^2+36x+4x^2-7]$

$=2(4x^2-7)^2[28x^2+36x-7]$

(v) $y=\frac{-7x}{x^2+6}$

$u=-7x$

$v=x^2+6$

$\frac{dy}{dx}=\frac{v.\frac{du}{dx}-u.\frac{dv}{dx}}{v^2}$

$=\frac{(x^2+6).-7-(-7x\times 2x)}{(x^2+6)^2}$

$=\frac{-7x^2-42+14x^2}{(x^2+6)^2}$

$=\frac{7x^2-42}{(x^2+6)^2}$

(b) Point is $x=1$, $y=\frac{1}{2\times 1}=\frac{1}{2}$

Gradient is $\frac{dy}{dx}$

$y=\frac{1}{2x}$

$=\frac{1}{2}x^{-1}$

$\frac{dy}{dx}=-\frac{1}{2}x^{-2}$

$=\frac{-1}{2x^2}$

At $x=1$, $\frac{dy}{dx}=\frac{-1}{2}$

Equation is $y-\frac{1}{2}=\frac{-1}{2}(x-1)$

$2y-1=-x+1$

$x+2y-2=0$

a) (i) $\cos(\alpha+\phi) = \cos\alpha\cdot\cos\phi - \sin\alpha\cdot\sin\phi$

ii) $\cos 2\phi = \cos\phi\cos\phi - \sin\phi\sin\phi$

$= \cos^2\phi - \sin^2\phi$

$= \cos^2\phi - (1-\cos^2\phi)$

$= 2\cos^2\phi - 1$

(iii) Let $\phi = 15^\circ$

$\cos 30^\circ = 2\cos^2 15^\circ - 1$

$\therefore \cos^2 15^\circ = \frac{\cos 30^\circ + 1}{2}$

$= \frac{\frac{\sqrt{3}}{2}+1}{2}$

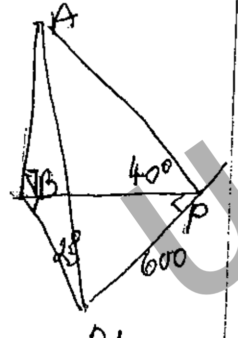
$= \frac{\frac{\sqrt{3}+2}{2}}{2}$

$= \frac{\sqrt{3}+2}{4}$

$\therefore \cos 15^\circ = \sqrt{\frac{\sqrt{3}+2}{4}}$

$= \frac{\sqrt{2+\sqrt{3}}}{2}$

(b)



Method: Find BQ interms of AB

Find BP interms of AB.

Use Pythagoras Theorem in $\triangle BQP$.

$\tan 65^\circ = \frac{BQ}{AB}$

$\therefore BQ = AB \times \tan 65^\circ$

$\tan 50^\circ = \frac{BP}{AB}$

$\therefore BP = AB \times \tan 50^\circ$

$BQ^2 = BP^2 + 600^2$

$\therefore AB^2\tan^2 65^\circ = AB^2\tan^2 50^\circ + 600^2$

$AB^2\tan^2 65^\circ - AB^2\tan^2 50^\circ = 600^2$

$AB^2(\tan^2 65^\circ - \tan^2 50^\circ) = 600^2$

$\therefore AB^2 = \frac{600^2}{(\tan^2 65^\circ - \tan^2 50^\circ)}$

$\therefore AB = \sqrt{\frac{600^2}{(\tan^2 65^\circ - \tan^2 50^\circ)}}$

$= 336.53\ldots$

$= 337$ metres to nearest metre.

(c) (i) $3\cos\phi + 4\sin\phi = R\cos(\phi-\alpha)$

where $R = \sqrt{3^2+4^2} = 5$

$\tan\alpha = \frac{4}{3}$

$\therefore \alpha = 53^\circ 8'$

$\therefore 3\cos\phi + 4\sin\phi = 5\cos(\phi - 53^\circ 8')$

(ii) $3\cos\phi + 4\sin\phi = 2 = 5\cos(\phi - 53^\circ 8')$

$\therefore \cos(\phi - 53^\circ 8') = \frac{2}{5}$

$\phi - 53^\circ 8' = 66^\circ 25'$ or $293^\circ 34'$

$\therefore \phi = 66^\circ 25' + 53^\circ 8'$ or $293^\circ 34' + 53^\circ 8'$

$= 119^\circ 33'$, $346^\circ 42'$

Question 6.

(a) Step 1. Find the x-value of intersection of $y=f(x)$ and $y=g(x)$.

Step 2. Find $f'(x)$ and $g'(x)$ at that x-value.

Step 3. Use $f'(x)$ as m_1, $g'(x)$ as m_2

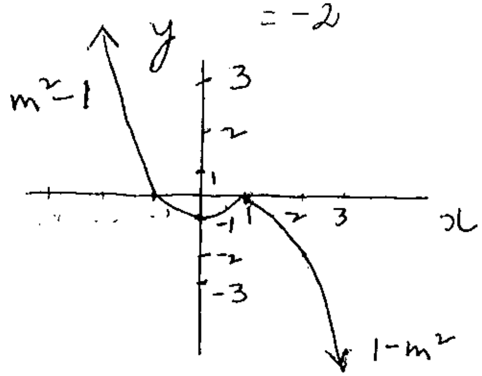
then $\tan\theta = \frac{m_1 - m_2}{1+m_1m_2}$ will find θ.

(b) $g(m) = \begin{cases} 1-m^2 & \text{if } m>1 \\ m^2-1 & \text{if } m\le 1 \end{cases}$

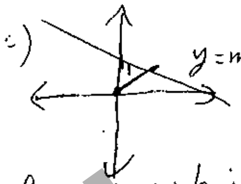
$g(2) + g(-1) - g(0) = (1-2^2) + ((-1)^2-1) - (0^2-1)$

$= -3 + 0 + 1$

$= -2$



(c)



If $y=mx+b$ is 1 unit from the origin then its perpendicular distance from $(0,0)$ is 1

$mx - y + b = 0$

$\therefore d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

$1 = \left|\frac{m\times 0 + -1\times 0 + b}{\sqrt{m^2+(-1)^2}}\right|$

$\therefore 1 = \frac{b}{\sqrt{m^2+(-1)^2}}$

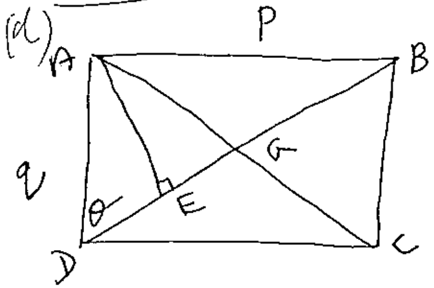
Square both sides

$1 = \frac{b^2}{m^2+1}$

$\therefore m^2+1 = b^2$

Question no. · continued

(d)



(i) In ΔDAB:

$\cos\theta = \dfrac{q}{DB}$

$= \dfrac{q}{\sqrt{p^2+q^2}}$

$DB^2 = q^2 + p^2$

$\therefore DB = \sqrt{q^2+p^2}$

(ii) In ΔADE

$\cos\theta = \dfrac{DE}{q}$

$DE = q \times \cos\theta$

$= q \times \dfrac{q}{\sqrt{p^2+q^2}}$

$= \dfrac{q^2}{\sqrt{p^2+q^2}}$

(iii) $EG = \frac{1}{2} BD - DE$

$= \frac{1}{2} \times \sqrt{q^2+p^2} - \dfrac{q^2}{\sqrt{p^2+q^2}}$

$= \dfrac{\frac{1}{2}\sqrt{p^2+q^2} \times \sqrt{q^2+p^2} - q^2}{\sqrt{p^2+q^2}}$

$= \dfrac{p^2+q^2-2q^2}{2\sqrt{p^2+q^2}}$

$= \dfrac{p^2-q^2}{2\sqrt{p^2+q^2}}$

$AE^2 = q^2 - DE^2$

$= q^2 - \left(\dfrac{q^2}{\sqrt{p^2+q^2}}\right)^2$

$= q^2 - \dfrac{q^4}{p^2+q^2}$

$= \dfrac{q^2(p^2+q^2) - q^4}{p^2+q^2}$

$AE^2 = \dfrac{q^2p^2}{p^2+q^2}$

$\therefore AE = \sqrt{\dfrac{q^2p^2}{p^2+q^2}}$

$= \dfrac{pq}{\sqrt{p^2+q^2}}$

$\therefore$ Area ΔAEG is

$\frac{1}{2} \times AE \times EG = \frac{1}{2} \times \dfrac{pq}{\sqrt{p^2+q^2}} \times \dfrac{p^2-q^2}{2\sqrt{p^2+q^2}}$

$= \dfrac{pq(p^2-q^2)}{4(p^2+q^2)}$