

ANSWERS QUESTION 1

Question 1 (a) (i)

Criteria	
Marks	1

• One mark for the answer

Answer: $\sqrt{32} = 4\sqrt{2}$

Question 1 (a) (ii)

Criteria	
Marks	1

• One mark for correct answer

Answer: $2x(1-x) - (x+2) = 2x - 2x^2 - x - 2$
 $= -2x^2 + x - 2$

Question 1 (b)

Criteria	
Marks	2

• One mark for 1.27586... and one for rounding

Answer: 1.27586... = 1.28(2 dp)

Question 1 (c)

Criteria	
Marks	2

• One mark for method, one for simplification

Answer: $x + 3y = 2 \dots A$
 $2x - y = 11 \dots B$
 $B \times 3 + A: 7x = 35$
 $\therefore x = 5; y = -1$

Question 1 (d)

Criteria	
Marks	2

• One mark for multiplying by conjugate, one for simplification

Answer: $\frac{3\sqrt{2} + 1}{2\sqrt{3} + 1} \times \frac{2\sqrt{3} + 1}{2\sqrt{3} + 1} = \frac{6\sqrt{6} + 3\sqrt{2} + 2\sqrt{3} + 1}{11}$

Question 1 (e)

Criteria	
Marks	2

• One mark for rearranging the numerator and one for simplification

Answer: $\frac{2x-1}{1-2x} - \frac{1}{1-2x} = -1$

ANSWERS QUESTION 2

Question 1 (i)

Criteria	
Marks	2

• One mark for $-4 < x + 2 < 4$, one for simplification

Answer: $|x+2| < 4$
 $-4 < x+2 < 4$
 $\therefore -6 < x < 2$

Question 2 (a) (i)

Criteria	
Marks	1

• One mark for the answer

Answer: $3m^2 - 30m = 3m(m - 10)$

Question 2 (a) (ii)

Criteria	
Marks	2

• One mark for taking out the common factor, one for simplification

Answer: $16y^2 - y = y(16y - 1) = y(4y - 1)(4y + 1)$

Question 2 (a) (iii)

Criteria	
Marks	2

• One mark for each factor

Answer: $8x^2 - 125y^2 = (2x - 5y)(4x^2 + 10xy + 25y^2)$

Question 2 (b)

Criteria	
Marks	2

• One mark for method, one for simplification

Answer: $\sin 300^\circ = \sin(360^\circ - 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

Question 2 (c)

Criteria	
Marks	3

• One mark changing the sine and tangent ratios and one for simplification

Answer: $\frac{\sin(180^\circ - \theta)}{\sin(90^\circ - \theta)} \times \tan(90^\circ - \theta) = \frac{\sin \theta}{\cos \theta} \times \cot \theta = \frac{\sin \theta}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} = 1$

Criteria	• One mark for $1000p=212.212\dots$ and one for simplification
Marks	2

Answer:

Let $p = 0.212$
 $\therefore p = 0.212212\dots$
 $1000p = 212.212\dots$
 $999p = 212$
 $p = \frac{212}{999}$

ANSWERS QUESTION 3

Criteria	• One mark for finding the gradient, one for simplification
Marks	2

Question 3 (a)

Answer:

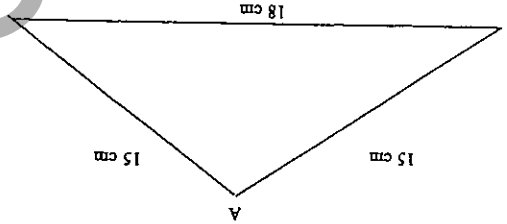
$2x + 6y + 3 = 0 \therefore \perp \text{ grad} = 3$
 eqn is $y - 3 = 3(x - 1)$

$\therefore 3x - y = 0$

Criteria	• One for $\cos A = \frac{126}{450}$ and one for simplification
Marks	2

Question 3 (b)

Answer:



$\cos A = \frac{15^2 + 15^2 - 18^2}{2(15)(15)} = \frac{126}{450}$
 $\therefore A = 73.44^\circ$

Question 3 (c)

Criteria	• One for factorisation, one for simplifying factors and one for taking the limit
Marks	3

Answer:

$\lim_{x \rightarrow -1} \frac{4x^2 + 3x - 1}{x^2 - 1} = \frac{4(-1)^2 + 3(-1) - 1}{(-1)^2 - 1} = \frac{0}{0}$
 $\lim_{x \rightarrow -1} \frac{(4x-1)(x+1)}{(x-1)(x+1)} = \lim_{x \rightarrow -1} \frac{4x-1}{x-1} = \frac{4(-1)-1}{-1-1} = \frac{-5}{-2} = \frac{5}{2}$

Criteria	• One mark for domain and one for the range
Marks	2

Question 3 (d)

Answer:

$y = \sqrt{x+5} \therefore \text{domain is } x+5 \geq 0 \therefore x \geq -5$
 range is $y \geq 0$

ANSWERS QUESTION 4

Criteria	• One mark for each gradient
Marks	3

Question 4 (a) (i)

Answer:

$M_{AB} = \frac{4}{5}, M_{BC} = \frac{4}{5}, M_{AC} = \frac{9}{1}$

Criteria	• One mark for the correct answer
Marks	1

Question 4 (a) (ii)

Answer:

$\frac{5}{4} \times \frac{4}{5} = 1 \therefore \text{a right angled triangle}$

Criteria	• One mark for correct answer
Marks	1

Question 4 (a) (iii)

Answer:

Midpoint = $\left(\frac{7-2}{2}, \frac{-1-2}{2} \right) = \left(\frac{5}{2}, -\frac{3}{2} \right)$

Criteria	• One mark for correct answer
Marks	1

Question 4 (a) (iv)

Answer:

$d = \sqrt{\left(2 - \frac{2}{2} \right)^2 + \left(3 + \frac{3}{2} \right)^2} = \frac{\sqrt{82}}{2} \therefore (= 4.528)$

Criteria	• One mark each for d_{AM} and d_{CM} and one for conclusion
Marks	3

Question 4 (a) (v)

Answer:

$d_{AM} = \sqrt{\left(-2 - \frac{2}{2} \right)^2 + \left(-2 + \frac{3}{2} \right)^2} = \frac{\sqrt{82}}{2}$
 $d_{MC} = \sqrt{\left(7 - \frac{2}{2} \right)^2 + \left(-1 + \frac{3}{2} \right)^2} = \frac{\sqrt{82}}{2}$

$\therefore d_{AM} = d_{CM} = d_{MC} = \frac{\sqrt{82}}{2}$

M is the centre of the circle through A, B, C

ANSWERS QUESTION 6

Criteria	Marks
• One mark for radius and centre of circle, one for perpendicular distance and one for conclusion	3

Answer:

(i) Centre of circle is $(2, -3)$ and radius of circle is $\frac{5}{4}$

(ii) \perp distance from centre to the line $d = \frac{|3(2) + 4(-3) + 10|}{\sqrt{3^2 + 4^2}} = \frac{5}{4}$ = the radius \therefore a tangent

ANSWERS QUESTION 5

Question 5 (a) (i)

In $\triangle ABC$ and $\triangle ABD$,

$\angle BDA = 180 - 90$ (angles on straight line)

$\angle DAB$ is common

$\therefore \angle BCA = \angle ABD$ (remaining angles in triangles - angle sum of a triangle)

$\therefore \triangle ABC \sim \triangle ABD$ (Angle-Angle-Angle test for similarity)

(ii) $\frac{AB}{AD} = \frac{BC}{BD}$
 $12.8 = \frac{16}{AD} \times 20 = 40 - AD$
 $12.8 = AD$

Criteria	Marks
• One mark for substitution into sine rule and one for simplification	2

(b)

$\frac{160}{AC} = \frac{\sin 60^\circ}{\sin 80^\circ}$

$\therefore AC = \frac{160 \sin 60^\circ}{\sin 80^\circ} = 140.7 \text{ miles} \approx 141 \text{ miles}$

Criteria	Marks
• One mark for dividing by $\cos x$ and one for simplification	2

Answer:

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\sin x} = 1$ where $\cos x \neq 0$

$\therefore \tan x = 1$

$x = 45^\circ, 225^\circ$

Answer:

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm \frac{\sqrt{2}}{2}$

$\therefore x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

Criteria	Marks
• One mark for any two angles and one for the remaining angles	2

(ii)

Question 6 (a)

(i) $\tan \theta \sqrt{1 - \sin^2 \theta} = \tan \theta \sqrt{\cos^2 \theta}$
 $= \tan \theta \cos \theta$
 $= \frac{\sin \theta}{\cos \theta} \times \cos \theta = \sin \theta$

(iii)

LHS = $\text{cosec}^2 \theta \cdot \cot^2 \theta$

$= \frac{1}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta}$

$= \frac{\sin^2 \theta}{1 - \cos^2 \theta}$

$= \frac{\sin^2 \theta}{\sin^2 \theta} = 1$

= RHS

(b)

$\frac{1}{3} (2-x) - 2(x+3) = 4$

$2-x-6(x+3) = 12$

$2-x-6x-18 = 12$

$-7x = 28$

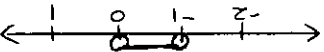
$x = -4$

Criteria	Marks
• One mark for the answer	1

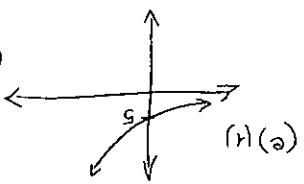
(c)

Answer:

$1 < 1-x < 2 \Rightarrow 0 < -x < 1 \Rightarrow -1 < x < 0$



(d) (ratio of intercepts)
 $\frac{1}{x} = \frac{5}{2}$
 $x = \frac{2}{5}$



$\frac{1}{2}$ mark for curve
 $\frac{1}{2}$ mark for intercept

(ii) When $x = 1.6$, $y = 15.16$ (2 d.p.)

2

2

2

2

2

ANSWERS QUESTION 8

ANSWERS QUESTION 8

(b) (i) Circumference of semi-circle = $\frac{1}{2}(2\pi r)$

But $r = x$

\therefore Circumference of semi-circle = πx

Lengths for rectangles = $6x + 6h$

\therefore Total length = $\pi x + 6x + 6h$

(ii) Area = area of semi-circle + 4 rectangles

= $\frac{1}{2}\pi x^2 + 4xh$

But $6h + \pi x + 6x = 36$

$\therefore h = \frac{36 - \pi x - 6x}{6}$

= $6 - \frac{\pi x}{6} - x$

$\therefore A = \frac{1}{2}\pi x^2 + 4x[6 - \frac{\pi x}{6} - x]$

= $\frac{\pi x^2}{2} + 24x - \frac{2\pi x^2}{3} - 4x^2$

= $\frac{\pi x - 4\pi x^2}{3} + 24x - 4x^2$

= $24x - 4x^2 - \frac{\pi x^2}{3}$

\therefore asymptote at $x = -\frac{2}{1}$

$x \neq -\frac{2}{1}$

$2x \neq -1$

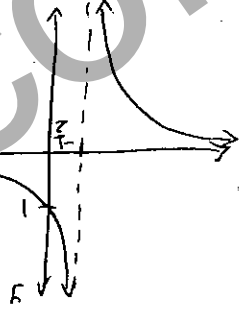
$1 + 2x \neq 0$

(c) $1 + 2x \neq 0$

$2x \neq -1$

$x \neq -\frac{2}{1}$

\therefore asymptote at $x = -\frac{2}{1}$



(d) $f(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 1 - (2x^2 - 1)}{h}$

= $\lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h}$

= $\lim_{h \rightarrow 0} \frac{4x + 2h}{1} = 4x$

(e) The function $f(x) = c$, where c is a constant, is a horizontal line when graphed. The gradient of any horizontal line is zero $\therefore f'(x) = 0$. Otherwise justification using derivative of a power rule i.e. $f(x) = cx^n$.

even

2

2

(a)

(i) $\frac{dy}{dx} = 12x^2$

(ii) $\frac{dy}{dx} = -5x^{-6} (= \frac{dc}{dt})$

(iii) $\frac{dy}{dx} = 2x$

(by either expanding, or using the product rule)

2

2

(iv) $y = \frac{x^2 - 16}{x^2 + 16}$

$\frac{dy}{dx} = \frac{(x^2 + 16)(2x) - (x^2 - 16)(2x)}{(x^2 + 16)^2}$

[note there is no (b) - oops!]

(c) $y = \sqrt{4x+1}$

$\therefore \frac{dy}{dx} = \frac{1}{2}(4x+1)^{-\frac{1}{2}} \times 4$

(d) $\frac{dy}{dx} = 2x - 3$

at $x = 3$, $\frac{dy}{dx} = 2 \times 3 - 3 = 3$

\therefore the gradient of the normal

at $x = 3$ is $-\frac{1}{3}$

$x + 3y - 6 = 0$

$x - 3 = -3y + 3$

$-\frac{1}{3}(x - 3) = y - 1$

formula:

\therefore The equation of the normal through (3, 1) is thus given by the point-gradient.

eqn is $y - 3 = \frac{3}{2}(x - 2)$

$0 = 2x - 3y + 5$

$2x - 3y + 5 = 0$

3