SOLUTIONS

Stella Maris College

Mathematics Extension 1

HSC Assessment 1 2005

Name:______________________________

Teacher:____________________________

General Instructions

- Working Time - 45 minutes
- Write using blue or black pen
- Board-approved calculators may be used
- All necessary working should be shown in the space provided

Total marks – 37

Weighting – 15%

Section	**Marks Achieved**	**Level of Axchievement** U=Unsatisfactory S=Satisfactory G=Good E=Excellent	**Teacher Comment**
Indefinite Integrals Q1, Q2	/6		
Definite Integrals Q3, Q4, Q6	/8		
Approximation Methods Q5, Q11(ii)	/4		
Areas Q7, Q8	/6		
Volumes Q9, Q10, Q11(i)	/7		
Integration by Substitution Q12, Q13	/6		

	Marks
1. Find a primitive function of $3x^2-2$. x^3-2x	1
2. Find: (i) $\int\sqrt{(t+1)}\,dt = \int (t+1)^{\frac{1}{2}}\,dt$ (1) (1) for + C $= \dfrac{(t+1)^{\frac{3}{2}}}{\frac{3}{2}}+C$ $= \dfrac{2(t+1)^{\frac{3}{2}}}{3}+C$ (1) (1)	3
(ii) $\int \dfrac{2-x^5}{x^3}\,dx = \int \dfrac{2}{x^3}-\dfrac{x^5}{x^3}\,dx$ $= \int(2x^{-3}-x^2)\,dx$ (1) $= \dfrac{2x^{-2}}{-2}-\dfrac{x^3}{3}+C$ $= -x^{-2}-\dfrac{x^3}{3}+C$ (1)	2
3. Evaluate: (i) $\int_0^2 y\,dy = \left[\dfrac{y^2}{2}\right]_0^2$ $= \left(\dfrac{4}{2}-0\right)$ $= 2$	2
(ii) $\int_{-1}^{1}(2x+1)^2\,dx = \left[\dfrac{(2x+1)^3}{3\times 2}\right]_{-1}^{1}$ $= \left[\dfrac{(2x+1)^3}{6}\right]_{-1}^{1}$ (1) $= \left(\dfrac{3^3}{6}\right)-\left(\dfrac{(-1)^3}{6}\right)$ $= \dfrac{27}{6}+\dfrac{1}{6}$	2

$= \dfrac{14}{3}$ (1)

4. Find the value of k if $\int_1^k (x+1)\,dx = 6$ such that $k>1$.	3

$\int_1^k (x+1)\,dx$

$= \left[\frac{x^2}{2}+x\right]_1^k$

$= \left(\frac{k^2}{2}+k\right)-\left(\frac{1}{2}+1\right)$ 1

$= \frac{k^2}{2}+k-\frac{3}{2} = 6$

$k^2+2k-3=12$

$k^2+2k-15=0$

$(k+5)(k-3)=0$

$k=-5$ or $k=3$ 1

but $k>1$

so no soln.

$\therefore\ k=3$ 1

5. Five values of the function $f(x)$ are shown in the table.	2

x	0	5	10	15	20
$f(x)$	12	20	25	27	23
	y_0	y_1	y_2	y_3	y_4

$h=5$

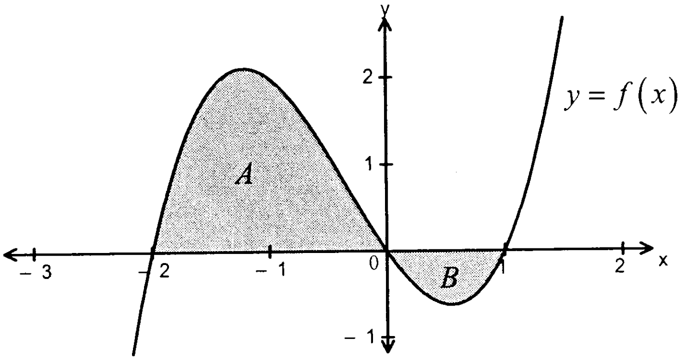
Use Simpson's Rule with the five function values given in the table to estimate $\int_0^{20} f(x)\,dx$.

$\int_0^{20} f(x)\,dx = \frac{h}{3}\left[y_0+y_n+4(y_1+y_3+\dots)+2(y_2+y_4+\dots)\right]$

$= \frac{5}{3}\left[12+23+4(20+27)+2(25)\right]$ 1

$= 455$ 1

6.	1



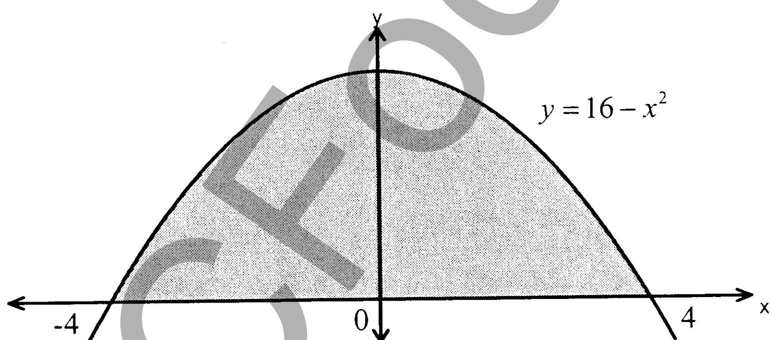
The diagram shows the graph of $y = f(x)$. The shaded areas are bounded by $y = f(x)$ and the x-axis.

The shaded area A is $2\frac{2}{3}$ square units and the shaded area B is $\frac{5}{12}$ square units.

Evaluate $\int_{-2}^{1} f(x)\, dx$. $= 2\frac{2}{3} - \frac{5}{12}$ ① for either subtraction or answer

$= 2\frac{1}{4}$

7.	2



The shaded region in the diagram is bounded by the curve $y = 16 - x^2$ and the x-axis. Calculate the **area** of the shaded region.

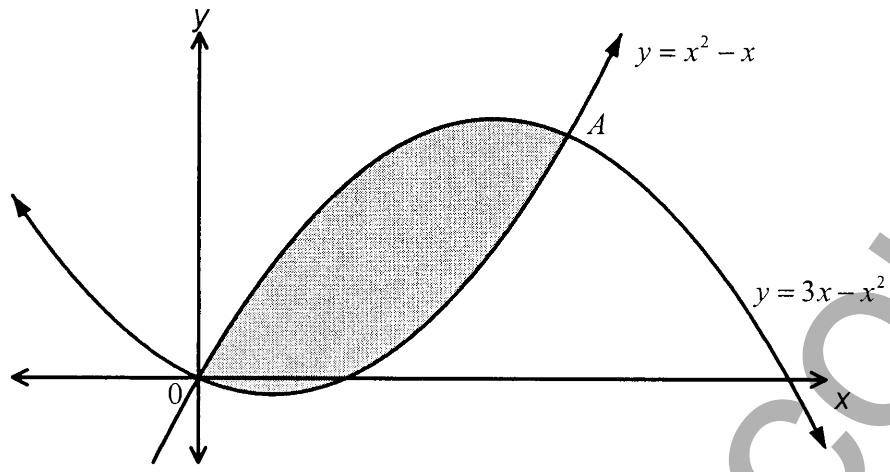
$A = 2\int_0^4 (16 - x^2)\, dx$ ① or $A = \int_{-4}^{4} (16 - x^2)\, dx$

$= 2\left[16x - \frac{x^3}{3}\right]_0^4$

$= 2\left[\left(16 \times 4 - \frac{64}{3}\right) - 0\right]$

$= 85\frac{1}{3}$ units^2 ①

8.



Ther graph of $y = 3x - x^2$ and $y = x^2 - x$ intersect at the points $(0,0)$ and A, as shown in the diagram.

(i) Find the x coordinate of the point A. **1**

$y = x^2 - x$ — ①

$y = 3x - x^2$ — ②

$x^2 - x = 3x - x^2$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0$ or $x = 2$

$\therefore$ $x = 2$ at A.

(ii) Find the **area** of the shaded region bounded by $y = 3x - x^2$ and $y = x^2 - x$. **3**

$A = \int_0^2 (3x - x^2) - (x^2 - x)\,dx$ ①

$= \int_0^2 (3x - x^2 - x^2 + x)\,dx$

$= \int_0^2 (4x - 2x^2)\,dx$

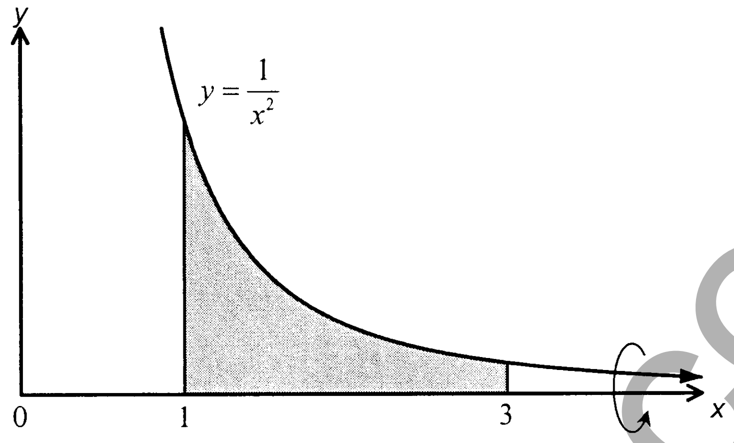
$= \left[\frac{4x^2}{2} - \frac{2x^3}{3}\right]_0^2$ ①

$= \left[2x^2 - \frac{2x^3}{3}\right]_0^2$

$= \left(8 - \frac{2}{3} \times 8\right) - 0$

$= 2\frac{2}{3}$ units^2 ①

9.



In the diagram the shaded region is bounded by the curve $y = \frac{1}{x^2}$, the x-axis and the lines $x = 1$ and $x = 3$.

The shaded region is rotated about the x-axis. Calculate the exact **volume** of the solid of revolution found. **3**

$$V = \pi \int_a^b y^2 \, dx$$

$$y = \frac{1}{x^2}$$

$$y^2 = \frac{1}{x^4} \quad 1$$

$$V = \pi \int_1^3 x^{-4} \, dx \quad 1$$

$$= \pi \left[\frac{x^{-3}}{-3} \right]_1^3$$

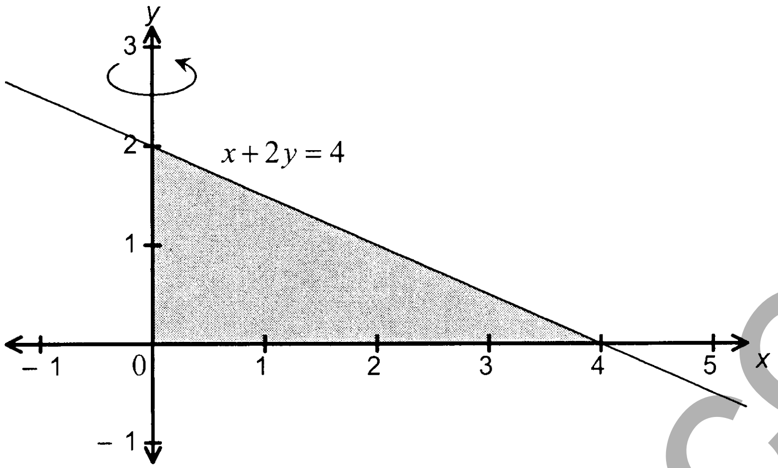
$$= \pi \left[-\frac{1}{3x^3} \right]_1^3$$

$$= \pi \left[\left(-\frac{1}{3 \times 3^3} \right) - \left(-\frac{1}{3} \right) \right]$$

$$= \pi \left[-\frac{1}{81} + \frac{1}{3} \right]$$

$$= \frac{26\pi}{81} \text{ units}^3 \quad 1$$

10.



In the diagram, the shaded region is bounded by the line $x + 2y = 4$ and the coordinate axes. The shaded region is rotated about the y-axis to form a cone.
Find, by integration, the exact **volume** of the cone. **3**

$V = \pi \int_a^b x^2 \, dy$

$x + 2y = 4$

$x = 4 - 2y$

$x^2 = (4-2y)^2$ (1)

$V = \pi \int_0^2 (4-2y)^2 \, dy$ (1)

$= \pi \left[\frac{(4-2y)^3}{3 \times -2} \right]_0^2$

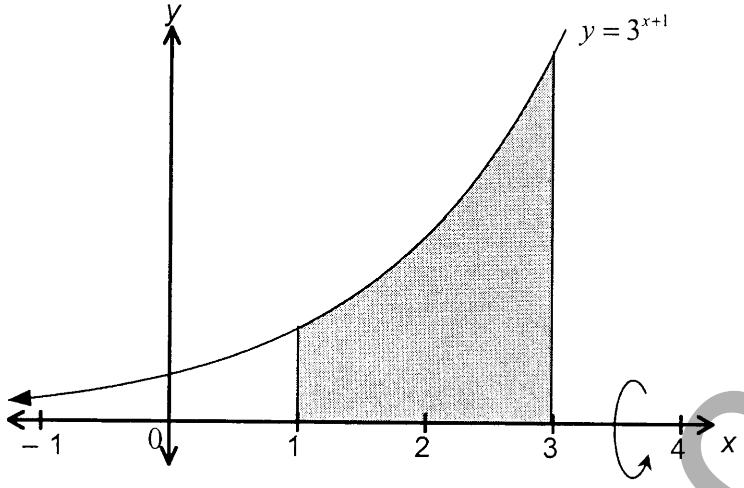
$= \pi \left[\frac{(4-2y)^3}{-6} \right]_0^2$

$= \pi \left[0 - \frac{4^3}{-6} \right]$

$= \frac{64\pi}{6}$

$= \frac{32\pi}{3}$ units^3 (1)

11.



The region under the curve $y = 3^{x+1}$ between $x = 1$ and $x = 3$ is rotated about the x-axis.

(i) State the integral that is required to evaluate the **volume** of the solid of revolution that is formed. **1**

$$\pi \int_1^3 y^2 \, dx = \pi \int_1^3 3^{2x+2} \, dx$$

$y = 3^{x+1}$

$y^2 = 3^{2x+2}$

(ii) Use the trapezoidal rule with three function values to find an approximaton of the volume of the solid. **2**

x	1	2	3
$f(x)$	3^4	3^6	3^8

$h = 1$

$$\pi \int 3^{2x+2} \, dx = \pi \frac{h}{2}\left[y_0 + y_2 + 2(y_1)\right]$$

$$= \pi \frac{1}{2}\left[3^4 + 3^8 + 2(3^6)\right] \quad (1)$$

$$= 4050\pi \text{ units}^3 \quad (1)$$

12. Using the substitution $u = x^2 + 1$, or otherwise, find $\int x\left(x^2+1\right)^{\frac{3}{2}} dx$.	3
$u = x^2+1$ $\frac{du}{dx} = 2x$ $du = 2x\,dx$ $\frac{1}{2}du = x\,dx$ $\int x(x^2+1)^{\frac{3}{2}}dx$ $= \frac{1}{2}\int u^{\frac{3}{2}}du$ $= \frac{1}{2}\frac{u^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + C$ $= \frac{1}{2}\times\frac{2}{5}u^{\frac{5}{2}} + C$ $= \frac{1}{5}u^{\frac{5}{2}} + C$ $= \frac{1}{5}(x^2+1)^{\frac{5}{2}} + C$	
13. Use the substitution $u = x^2 + 2x$ to evaluate $\int_1^2 \frac{x+1}{\sqrt[3]{x^2+2x}}\,dx$.	3
$u = x^2+2x$ $\frac{du}{dx} = 2x+2$ $du = 2(x+1)\,dx$ $\frac{1}{2}du = (x+1)dx$ when $x=2$, $u=8$ when $x=1$, $u=3$ $= \int_1^2 (x+1)(x^2+2x)^{-\frac{1}{3}}dx$ $= \frac{1}{2}\int_3^8 u^{-\frac{1}{3}}du$ $= \frac{1}{2}\left[\frac{u^{\frac{2}{3}}}{\frac{2}{3}}\right]_3^8$ $= \frac{3}{4}\left[4 - 3^{\frac{2}{3}}\right]$ $= \frac{3}{4}\left[4 - \sqrt[3]{9}\right]$ **END OF PAPER**	