

NSW Independent Trial HSC 2004
Mathematics Extension 1

Marking Guidelines

1a. Outcomes assessed: PE3, H3

Marking guidelines

Criteria	Marks
• states $\frac{5-x}{3-x} > 0$ as basic condition	1
• finds critical points at $x = 3$ and $x = 5$	1
• finds correct domain	1

Answer

$$\frac{5-x}{3-x} > 0 \text{ whence } x < 3 \text{ and } x > 5$$

1b. Outcomes assessed: i. P4 ii. Ask the Board of Studies

Marking guidelines

Criteria	Marks
• shows $x = 0$ gives same y value on each curve	1
• uses a valid method to find the angle	1
• finds the angle	1

Answer

i. $(0, 0)$ lies on both curves

ii. Gradient for $y = x^2 - x$ at $x = 0$ is -1 . Therefore angle is 45°

1c. Outcomes assessed: PE3

Marking guidelines

Criteria	Marks
• uses Factor theorem to set up equation	1
• solves to find answer	1

Answer

$$P(3) = 3^4 - 3 \times 3^3 + a \times 3^2 - a \times 3 - 12 = 0$$

$$6a - 12 = 0 \text{ so } a = 2$$

1d. Outcomes assessed: PE3

Marking guidelines

Criteria	Marks
• uses Tangent/Secant theorem to set up equation	1
• solves to correct solution	1

Answer

$$10^2 = x(x+15)$$

$$x^2 + 15x - 100 = 0$$

$$(x+20)(x-5) = 0$$

$$x = 5$$

1e. Outcomes assessed: PE3

Marking guidelines

Criteria	Marks
• uses appropriate result for circular permutations	1
• allows for the couple and calculates the answer	1

Answer

$$4 \times 2! = 48$$

2a. Outcomes assessed: HE6

Marking guidelines

Criteria	Marks
• finds dx and adjusts limits	1
• correctly substitutes and simplifies to integral of $\cos^2 \theta$	1
• finds correct integral and evaluates	1

Answer

$$x = 2 \sin \theta \Rightarrow dx = 2 \cos \theta d\theta$$

$$\text{If } x = 1, \theta = \frac{\pi}{6}; x = -1, \theta = \frac{-\pi}{6}$$

$$\text{Therefore, } I = \int_{-\pi/6}^{\pi/6} \sqrt{4 - 4 \sin^2 \theta} d\theta$$

$$= 4 \int_{-\pi/6}^{\pi/6} \cos^2 \theta d\theta$$

$$= 8 \left[\frac{1}{2} \left(\theta + \frac{1}{2} \sin 2\theta \right) \right]_{-\pi/6}^{\pi/6} = \frac{2\pi}{3} + \sqrt{3}$$

2b. Outcomes assessed: HE3

Marking guidelines

Criteria	Marks
• correct expansion for binomial expression	1
• calculates correct value of r	1
• calculates answer	1

Answer

$$\left(x - \frac{3}{x} \right)^8 = \sum_{r=0}^8 \binom{8}{r} x^r \left(\frac{-3}{x} \right)^{8-r} = \sum_{r=0}^8 \binom{8}{r} (-3)^{8-r} x^{2r-8} \Rightarrow 2r - 8 = 0 \Rightarrow r = 4$$

$$\text{Therefore, the term is } \binom{8}{4} (-3)^4 = 5670$$

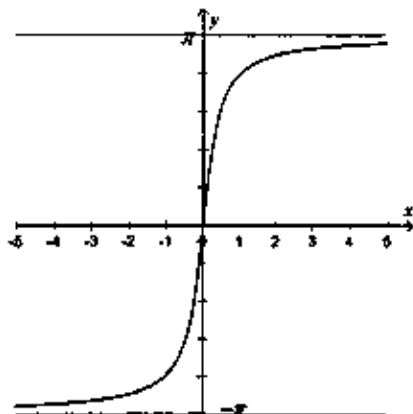
2c. Outcomes assessed: i. HE4 ii. HE4

Marking guidelines

Criteria	Marks
i. Correct graph shape with clearly marked axes	1, 1
ii. Correct domain and range	1

Answer

i.



ii.

X : x is real

Y : $-\pi \leq y \leq \pi$

2d. Outcomes assessed: ask the Board of Studies

Marking guidelines

Criteria	Marks
• chooses and applies an appropriate method	1
• obtains A and B (method 1) or $t = -\frac{1}{2}$ (method 2)	1
• correct answer	1

Answer

$$3 \cos \theta - 4 \sin \theta = A \cos(\theta + B)$$

$$= A \cos \theta \cos B - A \sin \theta \sin B$$

$$A \cos B = 3$$

$$A \sin B = 4$$

$$\Rightarrow A = 5; B = \tan^{-1}\left(\frac{4}{3}\right) \Rightarrow B = .927 \text{ rad}$$

$$5 \cos(\theta + B) = 5, -\pi \leq \theta \leq \pi$$

$$\theta + B = 0 \Rightarrow \theta = -0.93 \text{ rad}$$

OR:

$$\cos \theta = \frac{1-t^2}{1+t^2}; \sin \theta = \frac{2t}{1+t^2}$$

$$3 \times \frac{1-t^2}{1+t^2} - 4 \times \frac{2t}{1+t^2} = 5$$

$$8t^2 + 8t + 2 = 0$$

$$2(4t+1)(4t+1) = 0$$

$$\Rightarrow t = -\frac{1}{2} \Rightarrow \tan \frac{\theta}{2} = -\frac{1}{2}$$

$$\Rightarrow \frac{\theta}{2} = -.464 \text{ rad} \Rightarrow \theta = -0.93 \text{ rad}$$

3a. Outcomes assessed: PE3

Marking guidelines

Criteria	Marks
• correct answer	1

Answer

$${}^{20}C_8 = 125\,970$$

3b. Outcomes assessed: HE6

Marking guidelines

Criteria	Marks
• correctly replaces $\sin^2 2x$ with the appropriate result	1
• finds correct integral	1
• correct answer	1

Answer

$$\begin{aligned}\int_0^{\pi/12} \sin^2 2x \, dx &= \left[\frac{1}{2} \left(x - \frac{1}{4} \sin 4x \right) \right]_0^{\pi/12} \\ &= \frac{1}{2} \left(\frac{\pi}{12} - \frac{1}{4} \sin \frac{\pi}{3} \right) \\ &= \frac{\pi}{24} - \frac{\sqrt{3}}{16}\end{aligned}$$

3c. Outcomes assessed: HE2

Marking guidelines

Criteria	Marks
• Verifies the solution when $n = 1$	1
• Attempts to prove that if $S(n)$ is true, then $S(n+1)$ is true	1
• Correctly shows that if $S(n)$ is true, then $S(n+1)$ is true	1

Answer

$$S(n): 1 + 5 + 9 + \dots + 4n - 3 = 2n^2 - n$$

$$S(1): LHS = 1; RHS = 2 \times 1^2 - 1 = 1$$

$$S(k): 1 + 5 + \dots + 4k - 3 = 2k^2 - k$$

$$S(k+1): 1 + 5 + 9 + \dots + 4k - 3 + 4(k+1) - 3$$

$$= 2k^2 - k + 4k + 1$$

$$= 2k^2 + 3k + 1$$

$$= 2(k^2 + 2k + 1) - (k + 1)$$

$$= 2(k+1)^2 - (k+1)$$

Therefore, if $S(n)$ is true, then $S(n+1)$ is true.

But $S(1)$ is true, so $S(2)$ is true. Hence $S(3)$ is true and so on for all positive integer values of n

3d. Outcomes assessed: i. HE4 ii. HE1, HE4, HE7

Marking guidelines

Criteria	Marks
i. • knows and correctly sets up Newton's Method	1
• obtains correct answer	1
ii. • establishes $x_2 = -2x_1$	1
• concludes $ x_2 > x_1 $	1
• gives correct explanation	1

Answer

i. $f(x) = x^{1/3}$; $f'(x) = \frac{1}{3}x^{-2/3}$ ii. $x_2 = x_1 - \frac{x_1^{1/3}}{\frac{1}{3}x_1^{-2/3}}$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1 - \frac{1}{\frac{1}{3} \times 1} = -2$$

$$= -2x_1$$

$$\therefore |x_2| = |-2x_1| = 2|x_1|$$

$$\therefore |x_2| > |x_1|$$

Method fails because the approximations do not converge

4a. Outcomes assessed: i. HE3, HE4 ii. P4 iii. H5, P3

Marking guidelines

Criteria		Marks
i.	• finds first and second derivative	1
	• states $\ddot{x} = -9x$ so motion is SHM	1
ii.	• answer	1
iii.	• answer	1

Answer

i. $\dot{x} = -6\sin(3t + \pi/6)$
 $\ddot{x} = -18\cos(3t + \pi/6) = -9x$

Therefore, motion is Simple Harmonic

ii. $2\pi/3$

iii. $\dot{x} = -6\sin(3t + \pi/6) = 0$
 $3t + \pi/6 = 0, \pi, 2\pi$
 $3t = -\pi/6, 5\pi/6, 11\pi/6$
 $t = 5\pi/18$

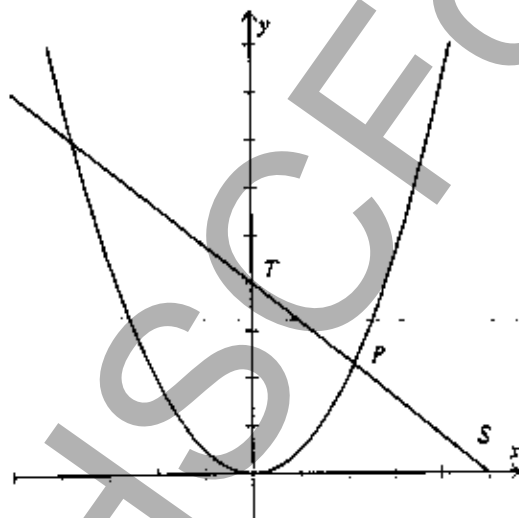
4b Outcomes assessed: i. P4 ii. PE4 iii. PE4

Marking guidelines

Criteria		Marks
i.	• correct diagram	1
ii.	• correct formula and coordinates of S and T	1,1,1
iii.	• correct values	1

Answer

i.



ii. $x + py = 2ap + ap^3$
 $T: x = 0 \Rightarrow y = 2a + ap^2$
 $S: y = 0 \Rightarrow x = 2ap + ap^3$

iii. If P is the midpoint of ST:

$$2ap = \frac{0 + (2ap + ap^3)}{2}$$

$$4ap = 2ap + ap^3$$

$$ap^3 - 2ap = 0$$

$$ap(p^2 - 2) = 0$$

$$p = 0, p = \pm\sqrt{2}$$

$$\therefore p = \pm\sqrt{2}$$

4c Outcomes assessed: i. H5 ii. HE3

Marking guidelines

Criteria		Marks
i.	• clear and correct explanation	1
ii.	• correctly uses Binomial Probability	1
	• answer	1

Answer

i. There are 2^3 possibilities.

The number of permutations of 2 heads and 1 tail is $3!/2! = 3$

Therefore, the probability is $\frac{3}{8}$

$$\begin{aligned} \text{ii. } P(X=r) &= {}^{10}C_r p^r q^{10-r} = {}^{10}C_r \left(\frac{3}{8}\right)^r \left(\frac{5}{8}\right)^{10-r} \\ P(X>1) &= 1 - \left[{}^{10}C_0 \left(\frac{3}{8}\right)^0 \left(\frac{5}{8}\right)^{10} + {}^{10}C_1 \left(\frac{3}{8}\right)^1 \left(\frac{5}{8}\right)^9 \right] \\ &= 0.936 \end{aligned}$$

5a. Outcomes assessed: i. HE3 ii. HE3

Marking guidelines

Criteria		Marks
i.	• correct demonstration	1
ii.	• finds B	1
	• finds k	1
	• answer	1

Answer

$$\begin{aligned} \text{i. } \frac{dT}{dt} &= Bke^{kt} & 80 &= 25 + 75e^{30k} \Rightarrow k = -0.0103 \\ &= kBe^{kt} & t &= 60 \Rightarrow T = 25 + 75e^{60 \times -0.0103} \\ &= k(T-S) & &= 65.33^\circ \\ & & &= 65 \end{aligned}$$

$$\text{ii. } 100 = 25 + Be^{k \times 0} \Rightarrow B = 75$$

5b Outcomes assessed: HE3

Marking guidelines

Criteria		Marks
•	derives results for vertical motion	1
•	derives results for horizontal motion and Cartesian form	1
•	substitutes parameters and reduces to equation in $\tan x$	1
•	uses quadratic formula to obtain angles	1
•	states range of values	1

Answer on next page

Horizontal Motion

$$\ddot{x} = 0$$

$$\dot{x} = C \quad \text{when } t = 0, \dot{x} = 25 \cos \alpha$$

$$\dot{x} = 25 \cos \alpha$$

$$x = 25t \cos \alpha + k, \quad \text{when } t = 0, x = 0$$

$$x = 25t \cos \alpha$$

Substitute

$$t = \frac{x}{25 \cos \alpha}$$

into $y \rightarrow$

$$\text{And when } x = 20, y = 15$$

$$15 = -\frac{400}{125} \sec^2 \alpha + 20 \tan \alpha + 2$$

$$13 = -\frac{16}{5} \sec^2 \alpha + 20 \tan \alpha$$

$$65 = -16(\tan^2 \alpha + 1) + 100 \tan \alpha$$

$$16 \tan^2 \alpha - 100 \tan \alpha + 81$$

$$\tan \alpha = \frac{100 \pm \sqrt{100^2 - 4 \cdot 16 \cdot 81}}{32}$$

$$\tan \alpha = 0.956, 5.29366$$

$$\alpha = 44^\circ, 79^\circ$$

$$44^\circ \leq \alpha \leq 79^\circ$$

Vertical Motion

$$\ddot{y} = -10$$

$$\dot{y} = -10t + c \quad \text{when } t = 0, \dot{y} = 25 \sin \alpha$$

$$\dot{y} = -10t + 25 \sin \alpha$$

$$y = -5t^2 + 25t \sin \alpha + k \quad \text{when } t = 0, y = 2$$

$$y = -5t^2 + 25t \sin \alpha + 2$$

$$y = -5 \left(\frac{x}{25 \cos \alpha} \right)^2 + 25 \cdot \frac{x}{25 \cos \alpha} \sin \alpha + 2$$

$$y = -\frac{x^2}{125} \sec^2 \alpha + x \tan \alpha + 2$$

5c. Outcomes assessed: i. HE3 ii. PE3

Marking guidelines

Criteria		Marks
i.	• correctly uses binomial theorem to expand expressions	1
	• equates coefficients on both sides to obtain answer	1
ii.	• answer	1

Answer

$$i. (1+x)^{n+3} = \binom{n+3}{0} + \binom{n+3}{1}x + \binom{n+3}{2}x^2 + \dots + \binom{n+3}{n+3}x^{n+3}$$

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n}x^n; (1+x)^3 = \binom{3}{0} + \binom{3}{1}x + \binom{3}{2}x^2 + \binom{3}{3}x^3$$

$$\text{Coefficient of } x^k \text{ on LHS is } \binom{n+3}{k}; \text{ on RHS: } \binom{n}{k} \binom{3}{0} + \binom{n}{k-1} \binom{3}{1} + \binom{n}{k-2} \binom{3}{2} + \binom{n}{k-3} \binom{3}{3}$$

$$\text{Hence: } \binom{n+3}{k} = \binom{n}{k} + 3 \binom{n}{k-1} + 3 \binom{n}{k-2} + \binom{n}{k-3}$$

$$ii. 3 < k < n$$

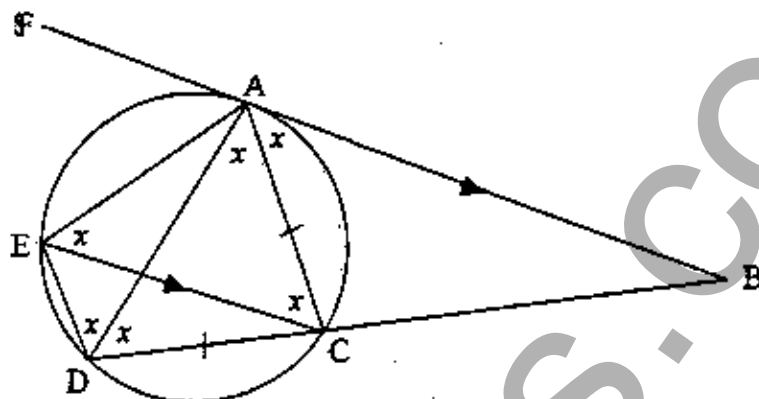
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6a. Outcomes assessed: PE3, PE6, HE7

Marking guidelines

Criteria	Marks
• correct use of appropriate geometrical theorems	1
• uses appropriate logical sequence to prove result	1
• uses correct setting out	1
• uses correct notation and terminology	1

Answer



Let $\angle BAC = x$

$$\angle AEC = \angle BAC = x$$
$$\angle ADC = \angle AEC = x$$
$$\angle ACE = \angle CAB = x$$
$$\angle ADE \cong \angle ACE = x$$
$$\angle DAC = \angle CDA = x$$

(The angle between a tangent and a chord is equal to the angle in the alternate segment)

(Angles in the same segment are equal (arc AC))

(Alternate angles $AB \parallel EC$)

(Angles in the same segment are equal (arc AE))

($\triangle ACD$ isosceles given $AC = DC$. $\therefore$ angles opposite equal sides are equal)

Since $\angle DAC = \angle ADE$ as both are equal to x , $AC \parallel ED$ since alternate angles are equal.

6b. Outcomes assessed: i. HES ii. HES iii. HES

Marking guidelines

Criteria		Marks
i.	• answer	1
ii.	• inverts integrand	1
	• integrates and finds c	1
	• rearranges to obtain expression for t	1
iii.	• answer	1

Answer

$$\begin{aligned} \text{i. } a &= v \frac{dv}{dx} = (2-x)^2 \times -2(2-x) \\ &= -2(2-x)^3 \end{aligned}$$

ii. $\frac{dx}{dt} = (2-x)^2 \Rightarrow \frac{dt}{dx} = (2-x)^{-2}$

$$\Rightarrow f = (2-x)^{-1} + c$$

$$t=0, x=0 \Rightarrow c = -1/2$$

$$t = \frac{1}{2-x} - \frac{1}{2} \Rightarrow x = 2 - \frac{2}{2t+1} = \frac{4t}{2t+1}$$

iii. $(2 - x)^2 = 1$

$$2-x=1 \text{ or } 2-x=-1$$

$x = 1.3$

But when $x = 3, f < 0$

So $x = 1$

6c. Outcomes assessed: i. HE4 ii. HE4

Marking guidelines

Criteria		Marks
i.	• makes statement about symmetry	1
	• specifies the line of symmetry is $y = x$	1
ii.	• gives a correct example	1

Answer

i. The function must be symmetrical about the line $y = x$

ii. Examples

7a. Outcomes assessed: i. PE5 ii. PE5, HE4

Marking guidelines

Criteria		Marks
i.	• uses appropriate procedures to find derivative of each term	1
	• answer	1
ii.	• adjusts functions to y-axis and sets up integral to find area	1
	• uses part i. result to obtain integral	1
	• evaluates integral	1

Answer

i. $\frac{d}{dx}(x \cos^{-1} x - \sqrt{1-x^2})$

$$= \cos^{-1} x + x \times \frac{-1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{1-x^2}} \times -2x$$

$$= \cos^{-1} x$$

ii. $A = \int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \cos^{-1} y dy$

$$= \left[y \cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$$

$$= \left[\frac{\sqrt{3}}{2} \times \frac{\pi}{6} - \sqrt{1-\frac{3}{4}} \right] - \left[\frac{1}{2} \times \frac{\pi}{3} - \sqrt{1-\frac{1}{4}} \right]$$

$$= \frac{\pi}{12}(\sqrt{3}-2) + \frac{\sqrt{3}-1}{2}$$

7b. Outcomes assessed: i. HE1 ii. PE2, PE6 iii. HE1, HE7

Marking guidelines

Criteria		Marks
i.	• answer	1
ii.	• finds both expressions for area in terms of m	1
	• finds values of m	1, 1
iii.	• finds both expressions for area in terms of n	1
	• constructs expression for ratio and simplifies	1
	• finds answers and justifies conclusion	1

Answer on next page

Answer

i. $0 \leq m \leq \frac{1}{2}$

ii. $P(1, m), Q(2, 2m)$

Area of trapezium, $APQD$, is $\frac{3m}{2}$

Area of $PBCQ$ is $1 - \frac{3m}{2}$

Ratio: either $\frac{\frac{3m}{2}}{1 - \frac{3m}{2}} = \frac{2}{1} \Rightarrow m = \frac{4}{9}$

or $\frac{\frac{3m}{2}}{1 - \frac{3m}{2}} = \frac{1}{2} \Rightarrow m = \frac{2}{9}$

iii. Now $\frac{1}{2} \leq n \leq 1$

$S(1, n), T(\frac{1}{n}, 1)$

Area of triangle SBT is $(1-n)(\frac{1}{n}-1)$

Area of remainder is $1 - (1-n)(\frac{1}{n}-1)$

Ratio: $\frac{\frac{1}{n}-2+n}{1 - (\frac{1}{n}-2+n)} = \frac{1-2n+n^2}{-1+3n-n^2}$

$\therefore \frac{1-2n+n^2}{-1+3n-n^2} = \frac{1}{2}$

$3n^2 - 7n + 3 = 0$

$n = \frac{7 \pm \sqrt{49 - 4 \times 3 \times 3}}{2 \times 3} = \frac{7 \pm \sqrt{13}}{6}$

$n = 1.7676, 0.5657$

Both are outside the limit above so k cannot divide the square in the ratio 2:1

The Trial HSC examination, marking guidelines /suggested answers and 'mapping grid' have been produced to help prepare students for the HSC to the best of our ability.

Individual teachers/schools may alter parts of this product to suit their own requirements.