Independent Schools

Independent Schools Trial Examination
2004 Mathematics Extension 1

HSC SUPPORT

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04 IS **1a** Find the domain of the function $f(x) = \log_e\left(\dfrac{5-x}{3-x}\right)$. **3**

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04 IS **1b** Consider the curves $y = x^3$ and $y = x^2 - x$.

(i) Show that the curves intersect at the point where $x = 0$. **1**

(ii) Find the acute angle between the curves at this point. **2**

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04 IS **1c** If $P(x) = x^4 - 3x^3 + ax^2 - ax - 12$ is divisible by $(x - 3)$, find the value of a. **2**

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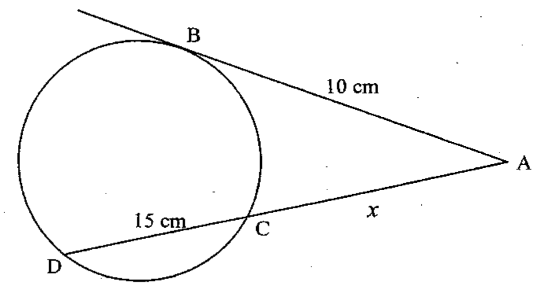
04 IS **1d**



In the diagram, AB = 10 cm, CD = 15 cm and AC = x cm. Find the value of x. **2**

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04 IS **1e** Six people are to be placed around a circular table. Two of them want to sit together. In how many ways can the table be arranged? **2**

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04 IS **2a** Find the exact value of $\displaystyle\int_{-1}^{1} \sqrt{4-x^2}\ dx$, using the substitution $x = 2\sin\theta$. **3**

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04 IS **2b** For the expansion of the expression $(x - \frac{3}{x})^8$, find the term independent of x. **3**

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04 IS **2c** (i) Sketch the graph of $y = 2\tan^{-1} 3x$. **2**

(ii) State the domain and range of the function. **1**

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04 IS **2d** Solve the equation $3\cos\theta - 4\sin\theta = 5$, for $-\pi \le \theta \le \pi$. **3**

Express your answers correct to 2 decimal places.

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04 IS **3a** In how man ways can 8 prefects be chosen from a group of 20 nominees? **1**

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04 IS **3b** Find the exact value of $\displaystyle\int_{0}^{\frac{\pi}{12}} \sin^2 2x\ dx$. **3**

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04 IS **3c** Use Mathematical Induction to show that $\displaystyle\sum_{r=1}^{n} 4r - 3 = 2n^2 - n$. **3**

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04 IS **3d** Jaime wants to use Newton's Method to obtain the zero of $\sqrt[3]{x} = 0$.

(i) Using Newton's Method once with a first approximation of $x = 1$, obtain a second approximation. **2**

(ii) Using Newton's Method with a first approximation of $x_1 \neq 0$, show that the **3**

second approximation, x_2, is such that $|x_2| > |x_1|$.
Explain the significance of this result.

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04 IS **4a** A particle is moving so that its displacement, x cm, at any time, t seconds, is given by the equation $x = 2\cos(3t + \frac{\pi}{6})$.

(i) Show that the particle movies in Simple Harmonic Motion. **2**

(ii) State the period of the motion. **1**

(iii) When does the particle first come to rest after $t = 0$? **1**

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04 IS **4b** P($2ap$, ap^2) is a point on the parabola $x^2 = 4ay$. The normal at P cuts the x axis at S and the y axis at T.

(i) Draw a half page sketch to show this information. **1**

(ii) State the equation of the normal to the parabola at P and hence show that S is the point ($ap(2 + p^2)$, 0) and that T is the point (0, $a(2 + p^2)$). **3**

1

(iii) Find the value(s) of p such that P is the midpoint of ST.

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04 IS **4c** (i) Explain why the probability of obtaining 2 heads and a tail when three coins are tossed is $\frac{3}{8}$. **1**

(ii) Sian tosses three coins 10 times in a row. Calculate the probability of obtaining 2 heads and a tail at least 2 times. Give your answer correct to 3 significant figures. **2**

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04 IS **5a** The rate at which a body cools in air is proportional to the difference between the temperature, T, of the body and the constant surrounding temperature, S. This can be expressed as $\frac{dT}{dt} = k(T - S)$, where t is in minutes and k is a constant.

(i) Show that $T = S + Be^{kt}$, where B is a constant, is a solution of the above equation. **1**

(ii) If a particular body cools from 100° to 80° in 30 minutes, find the temperature of the body after a further 30 minutes, given the surrounding temperature remains constant at 25°. Give your answer to the nearest degree, **3**

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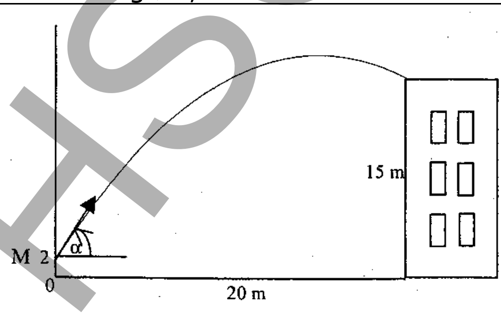
04 IS **5b**



A man of height 2 metres throws a ball from M to the roof of a 15 metre high building. He throws the ball at an initial velocity of 25 m/s, and he is 20 m from the base of the building.

Between which two angles of projection must he throw the ball to ensure it lands on the roof of the building? **5**

(Assume $\ddot{x} = 0$ and $\ddot{y} = -10$.)

04 IS **5c** (i) By considering $(1 + x)^{n+3} = (1 + x)^{n} (1 + x)^{3}$, show that **2**

$$\binom{n+3}{k} = \binom{n}{k} + 3\binom{n}{k-1} + 3\binom{n}{k-2} + \binom{n}{k-3}$$

(ii) Between what values must k lie? **3**

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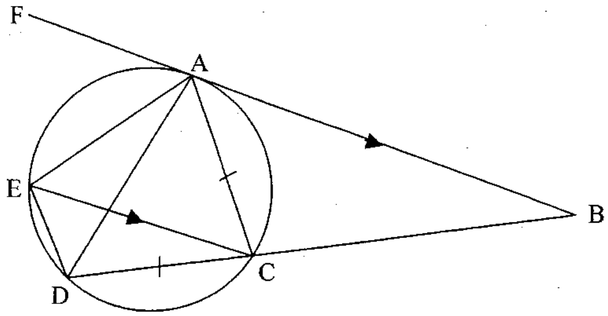
04 IS **6a**



AB is a tangent to the circle.
AB || EC and CD = AC.
Copy the diagram into your booklet.
Prove that AC || ED.

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04 IS **6b** A particle is moving in a straight line. At time t seconds, it has displacement x metres from a fixed point O on the line, velocity v ms^{-1} and acceleration a ms^{-2}. The particle starts from O and you are given that $v = (2 - x)^2$.

(i) Find an expression for a in terms of x. **1**

(ii) Find an expression for x in terms of t. **3**

(iii) Find the distance from O when the particle has a speed of 1 ms^{-1}. **1**

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04 IS **6c** (i) Given a function, $y = \text{f}(x)$, under what geometrical conditions would $\text{f}(x) = \text{f}^{-1}(x)$? **2**

(ii) Give an example of a function for which $\text{f}(x) = \text{f}^{-1}(x)$. **1**

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04 IS **7a** (i) Find $\dfrac{d}{dx}(x\cos^{-1}x - \sqrt{1-x^2})$. **2**

(ii) Find the area between the curve $y = \cos x$, the y axis and the lines $y = \dfrac{1}{2}$ and $y = \dfrac{\sqrt{3}}{2}$. **3**

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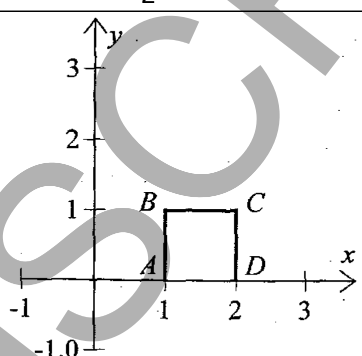
04 IS **7b**



The diagram shows a unit square, ABCD, where A(1, 0), B(1, 1), C(2, 1), D(2, 0). Copy the diagram into your workbook.

(i) A line, l, passing through the origin with gradient m, cuts the sides AB and CD at P and Q respectively. Comment on the possible values of m. **1**

(ii) For what value(s) of m does the line, l, divide the area of the square in the ratio 2:1? **3**

(iii) Another line, k, passes through the origin with gradient, n, and cuts the square through sides AB and BC at S and T respectively. Show that it is not **3** possible for k to divide the area of the square in the ratio 2:1.

A **1a.** $x < 3$ and $x > 5$ **1b.(i)** 45^o **1c.** $a = 2$ **1d.** $x = 5$ **1e.** $4! \times 2! = 48$ **2a.** $\frac{2\pi}{3} + \sqrt{3}$ **2b.** 5670 **2c.(ii)** dom: all real x range: $y = -\pi \le x \le \pi$ **2d.** -0.93 radians **3a.** ${}^{20}C_8 = 125\,970$ **3b.** $\frac{\pi}{24} - \frac{\sqrt{3}}{16}$ **3d.(i)** -2 **(ii)** approximations do not converge **4a.(ii)** $\frac{2\pi}{3}$ **(iii)** $\frac{5\pi}{18}$ **4b.(ii)** $x = 2ap + ap^3$ **(iii)** $p = \pm 2$ **4c.** 0.936 **5a.(ii)** 65^o **5b.(ii)** $45^o \le \alpha \le 79^o$ **5c.(ii)** $3 < k < n$ **6b.(i)** $a = -2(2 - x)^3$ **(ii)** $\frac{4t}{2t+1}$ **(iii)** $x = 1$ **6c.(i)** symmetrical about $y = x$. **7a.(i)** $\cos^{-1} x$ **(ii)** $\frac{\pi}{12}(\sqrt{3} - 2) + \frac{\sqrt{3}-1}{2}$ **7b.(i)** $0 \le x \le \frac{1}{2}$ **(ii)** 4:9, 2:9