2002 *Independent Schools*

Independent Schools Trial Examination

Mathematics Extension 1 Course

HSC SUPPORT

02 IS 1a (i) Sketch the graph of $y = x^3 - 4x$. **1**

(ii) Hence, or otherwise, solve $x^3 - 4x > 0$. **2**

02 IS 1b Find the coordinates of the point which divides the interval joining (2, 4) and (-1, -2) externally in the ratio 2:1. **3**

02 IS 1c Find the exact value of $\cos 15^\circ$. **2**

02 IS 1d (i) Sketch the graph of $y = \tan x$ for $0 \le x \le \pi$. **1**

(ii) Hence or otherwise, find values of x ($0 \le x \le \pi$.) such that the series $1 + \sqrt{3}\tan x + 3\tan^2 x + 3\sqrt{3}\tan^2 x + \ldots$ has a limiting sum. **3**

02 IS 2a Use the identity $\sin 2x = 2\sin x \cos x$ to find $\int_0^{\frac{\pi}{6}} \sin^2 x \cos^2 x \, dx$. **3**

02 IS 2b Evaluate $\int_0^1 \frac{4x}{(4x+1)^2} \, dx$, using the substitution $u = 4x + 1$. **3**

02 IS 2c A, B, C and D are points on the circumference of a circle. AB produced intersects CD produced at a point P. AB = 13 cm, BP = 3 cm and CD = 8 cm.

(i) Draw a clear sketch showing the above information. **1**

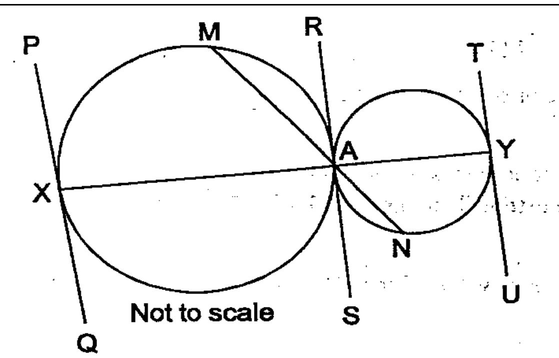
(ii) Find the length of DP. **2**

02 IS 2d Given $P(x) = x^3 - ax^2 + 4$,

(i) find a, if $x + 1$ is a factor of $P(x)$. **1**

(ii) Hence, write $P(x)$ in terms of its linear factors **2**

02 IS 3a Two circles touch at the point A. Lines through A meet at X and Y and at M and N respectively, as shown. RS, the tangent at A is shown. Copy the diagram into your workbook. Prove that the tangents at X and Y are parallel. **4**



02 IS 3b In how many ways can a jury of 7 people reach a majority decision? (a majority decision is one to which the majority agree.) **3**

02 IS 3c Use the Principle of Mathematical Induction to prove that $5^n > 3^n + 2^n$ for integers $n > 1$. **5**

02 IS 4a (i) Show that $x^3 - 3x + 1 = 0$ has a root between $x = 1$ and $x = 2$. **3**

(ii) Using $x = 1.5$ as a first approximation, obtain a better approximation of the root using Newton's Method once. [Answer to 2 decimal places]

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02 IS **4b** Use $(1 + x)^6 = (1 + x)(1 + x)^5$ to show that $\binom{6}{3} = \binom{5}{3} + \binom{5}{2}$. **3**

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02 IS **4c** $P(2ap, ap^2)$ is any point on the parabola $x^2 = 4ay$. The line k is parallel to the tangent at P and passes through the focus, S, of the parabola.

(i) Find the equation of the line k. **2**

(ii) The line k intersects the x-axis at the point Q. **2**
Find the coordinates of the midpoint, M, of the interval QS.

(iii) What is the equation of the locus of M? **1**

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02 IS **5a** The acceleration, a ms^{-2}, of a particle moving in a straight line is given by the equation, $a = \frac{x^3}{8} + \frac{x}{8}$, where x is the displacement in metres of the particle from the origin. v is the velocity of the particle at any time, t.

(i) If $v = \frac{1}{4}$ when $x = 0$, show that $v^2 = \frac{(1+x^2)^3}{16}$. **3**

(ii) If $x = 1$ when $t = 0$, find an expression for the displacement of the particle in terms of t. **3**

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02 IS **5b** A population of marsupials has an initial population of 500. Factors which influence the population include birthrates, the number of marsupials killed by feral animals, the amount of feed and so on.
The change in population, N, is given by the formula $N = \frac{500}{1 + ke^{-1.5t}}$,

where k is a constant and t is in months.

(i) Explain why the population will eventually die out. **1**

(ii) If at $t = 0$, the change in population is 1, use the formula to find how long it will take for only 100 marsupials to remain. Give your answer to the nearest month. **3**

(iii) Show that $\frac{dN}{dt} = \frac{3N}{1000}(500 - N)$ **2**

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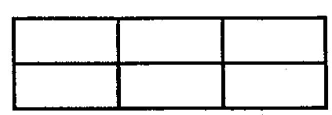
02 IS **6a** In how many different ways can 3 black and 3 white tiles be placed in the following grid? **2**



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02 IS **6b** The velocity, v ms^{-1}, of a particle moving along the x-axis in simple harmonic motion is given by $v^2 = 21 - 4x - x^2$, where x is the position of the particle.

(i) Between which two points on the x-axis does the particle oscillate? **1**

(ii) Find an expression for the acceleration, a ms^{-2}, in terms of x. **2**

(iii) What is the maximum velocity of the particle? **1**

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02 IS **6c** An object is projected at an initial velocity V ms^{-1}, from the ground level at an angle of θ to the horizontal. Use $g = 10$ ms^{-2}.

(i) Show that the horizontal and vertical components of the position of the particle are given by $x = Vt\cos\theta$ and $y = -5t^2 + Vt\sin\theta$. **2**

(ii) Derive an expression for the Cartesian equation for the motion. (ie. find y in terms of x). **2**

(iii) A particle is projected from ground level with an intial velocity of 80 ms^{-1}. It just clears a 2 metre high wall 25 metres from the point of projection. The base of the wall is at the same level as the point of projection. Calculate the angel(s) of projection to the nearest minute. **2**

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02 IS 7a Find $\int_0^{\frac{1}{4}} \frac{4}{\sqrt{1-4x^2}}\,dx$. **2**

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02 IS 7b A machine produces electronic components for computers. Sampling shows that the probability of a particular component being faulty is 8%. In a random sample of 20 components, what is the probability that:

(i) exactly 1 component is faulty? Give your answer to 3 decimal places. **1**

(ii) less than 3 components are faulty? Give your answer to 3 decimal places. **2**

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02 IS 7c (i) If $x = a + b$ and $y = a - b$, show that $a = \frac{x+y}{2}$ and $b = \frac{x-y}{2}$. **1**

(ii) Show that $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$. **2**

(iii) Hence find all solutions to $\cos 4\theta + \cos 3\theta + \cos 2\theta + \cos\theta = 0$ for $0 \le \theta \le 2\pi$. **4**

A **1a.(ii)** $-2 < x < 0$ and $x > 2$ **1b.** (-4, 8) **1c.** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **1d.(ii)** $0 < x < \frac{\pi}{6}$, $\frac{5\pi}{6} < x < \pi$

2a. $\frac{1}{8}\left[\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right]$ **2b.** $\frac{1}{20}(5\ln 5 - 4)$ **2c.(ii)** $x = 4$ **2d.(i)** a = 3 **(ii)** $P(x) = (x+1)(x-2)(x-2)$

3b. $\binom{7}{7} + \binom{7}{6} + \binom{7}{5} + \binom{7}{4} = 64$ **4a.(ii)** 1.53 **4c.(i)** $y = px + a$ **(ii)** $M\left(-\frac{p}{2a}, \frac{a}{2}\right)$ **(iii)** $y = \frac{a}{2}$

5a.(ii) $x = \tan\left[\frac{1}{4}(t + \pi)\right]$ 65° **5b.(i)** as $t \to 0$, $ke^{-1.5t} \to 0$, $N \to 500$ **(ii)** 5 months **6a.** 20

6b.(i) $x = -7$ and 3 **(ii)** $a = -2 - x$ **(iii)** $v = 5$ **6c.(ii)** $y = x\tan\theta - \frac{5x^2}{V^2}(1 + \tan^2\theta)$

(iii) 85° 30′, 4° 33′ **7a.** $\frac{\pi}{3}$ **7b.(i)** 0.328 **(ii)** 0.788 **7c.(iii)** $\frac{\pi}{3}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}$