CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2002
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Marking guidelines/ solutions

2603 - 2

Mathematics Extension I CSSA HSC Trial Examination 2002
Marking Guidelines

Question 1

1(a) **Outcomes Assessed: H5, PE5**

Marking Guidelines

Criteria	Marks
• finding first derivative	1
• finding second derivative in form $\dfrac{e^x}{(e^x+1)^2}$	1

Answer

$$\frac{d}{dx}\ln(e^x+1)=\frac{e^x}{e^x+1} \qquad \frac{d^2}{dx^2}\ln(e^x+1)=\frac{e^x.(e^x+1)-e^x.e^x}{(e^x+1)^2}=\frac{e^x}{(e^x+1)^2}$$

1(b) **Outcomes Assessed: H5, PE3, PE6**

Marking Guidelines

Criteria	Marks
• interpreting Σ notation to write sum of terms in expanded form	1
• calculating value of sum as $-\frac{5}{8}$	1

Answer

$$\sum_{k=1}^{4}\frac{(-1)^k}{k!}=-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}=\frac{-24+12-4+1}{24}=-\frac{5}{8}$$

1(c) **Outcomes Assessed: (i) P3 (ii) P3, PE2, HE7**

Marking Guidelines

Criteria	Marks
(i) • writing expressions for $1\pm\cos 2x$ in terms of $\cos^2 x$, $\sin^2 x$	1
• simplifying to obtain final result	1
(ii) • substituting $x=22\frac{1}{2}^\circ$ and $\cos 45^\circ=\frac{1}{\sqrt{2}}$ to find expression for $\tan^2 22\frac{1}{2}^\circ$	1
• using expression for $\tan^2 22\frac{1}{2}^\circ$ to show $\tan 22\frac{1}{2}^\circ=\sqrt{2}-1$	1

Answer

(i) $$\frac{1-\cos 2x}{1+\cos 2x}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$$

(ii) $$\tan^2 22\tfrac{1}{2}^\circ=\frac{1-\cos 45^\circ}{1+\cos 45^\circ}=\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}=\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$\tan^2 22\tfrac{1}{2}^\circ=\frac{(\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}=\frac{(\sqrt{2}-1)^2}{2-1}$$

$$\therefore\ \tan 22\tfrac{1}{2}^\circ=(\sqrt{2}-1),\quad \text{since}\quad \tan 22\tfrac{1}{2}^\circ>0$$

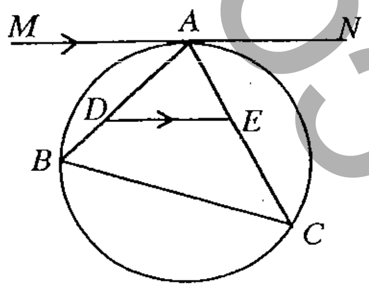
1(d) **Outcomes Assessed:** (i) (ii) **PE3** (iii) **H5, PE2, PE3**

Marking Guidelines

Criteria	Marks
(i) • copying diagram	0
(ii) • using alternate segment theorem	1
(iii) • using equal alternate angles with parallel lines to deduce $A\hat{D}E = M\hat{A}D$	1
• deducing $A\hat{D}E = E\hat{C}B$ with explanation	1
• deducing $BCED$ is cyclic by applying appropriate test	1

Answer

(i)



(ii)

$M\hat{A}B = A\hat{C}B$ (*angle between tangent MAN and chord AB equal to angle in alternate segment*)

(iii)

$A\hat{D}E = M\hat{A}D$ (*Alternate angles equal*, $DE \parallel MAN$)

$A\hat{D}E = E\hat{C}B$ (*Both equal to* $M\hat{A}D$)

$\therefore$ $BCED$ is a cyclic quadrilateral

(*Exterior angle* $A\hat{D}E$ = *opposite interior angle* $E\hat{C}B$)

Question 2

2(a) **Outcomes Assessed: P4**

Marking Guidelines

Criteria	Marks
• finding the x coordinate of P	1
• finding the y coordinate of P	1

Answer

$x = \dfrac{4\times 4 + 1\times(-2)}{4+1} = 2\cdot 8$, $y = \dfrac{4\times(-5)+1\times 3}{4+1} = -3\cdot 4$ $\therefore P(2\cdot 8,\ -3\cdot 4)$

2(b) **Outcomes Assessed: PE3**

Marking Guidelines

Criteria	Marks
• using at least one of the factors ${}^7C_2$, 3^5	1
• completing the calculation ${}^7C_2 \times 3^5 = 5103$	1

Answer

Choose the 2 questions to be answered correctly ${}^7C_2$ ways

Each of the 5 questions answered incorrectly can be answered in 3 ways. $\therefore$ ${}^7C_2 \times 3^5 = 5103$ ways

2(c) **Outcomes Assessed:** (i) **PE3** (ii) **PE3, PE6**

Marking Guidelines

Criteria	Marks
(i) • partial factorisation $P(x)=(x-1)(x^2+x-2)$	1
• completing factorisation $P(x)=(x+2)(x-1)^2$	1
(ii) • deducing $x \le -2$	1
• including $x=1$	1

Answer

(i)

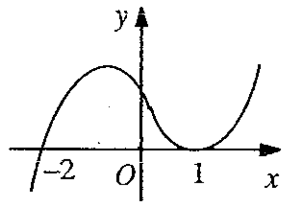
$(x-1)$ is a factor of $P(x)$

$x^3-3x+2=(x-1)(x^2+x-2)$

$=(x-1)(x-1)(x+2)$

$\therefore P(x)=(x+2)(x-1)^2$

(ii)



$y=(x+2)(x-1)^2$

By inspection of the graph,

$x^3-3x+2 \le 0$ when

$x \le -2$ or $x=1$

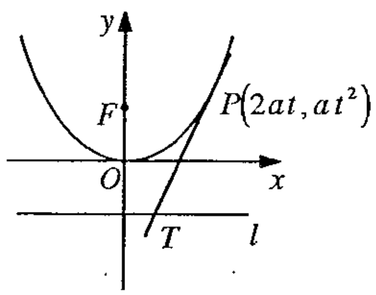
2(d) **Outcomes Assessed:** (i) **PE3** (ii) **PE3, PE4**

Marking Guidelines

Criteria	Marks
(i) • finding the x coordinate of T	1
(ii) • finding the gradient of PF	1
• finding the gradient of TF	1
• showing the product of the gradients is -1 to prove $TF \perp PF$	1

Answer

(i)



At T, $\left.\begin{array}{l} y=-a \\ tx-y-at^2=0 \end{array}\right\} \Rightarrow \begin{array}{l} tx=a(t^2-1) \\ x=a\left(t-\frac{1}{t}\right) \end{array}$

$\therefore T\left(a\left(t-\frac{1}{t}\right), -a\right)$

(ii) $F(0,a) \Rightarrow$ *gradient* $PF=\dfrac{a(t^2-1)}{2at}=\dfrac{1}{2}\left(t-\dfrac{1}{t}\right)$ and *gradient* $TF=\dfrac{-2a}{a\left(t-\frac{1}{t}\right)}=-\dfrac{2}{\left(t-\frac{1}{t}\right)}$

$\therefore$ *gradient PF . gradient TF* $=-1$ and hence $TF \perp PF$.

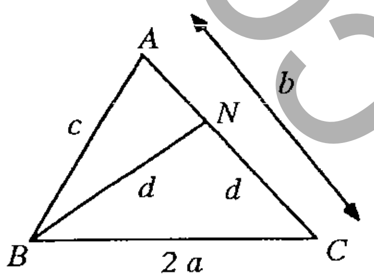
Question 3

(a) **Outcomes Assessed:** (i) **H5** (ii) **P4**

Marking Guidelines

Criteria	Marks
(i) • using similarity and sides in proportion to deduce $\frac{d}{2a}=\frac{c}{b}=\frac{b-d}{c}$	1
• selecting the appropriate relationships to show $bd=2ac$, $c^2=b(b-d)$	1
• using these simultaneously to show $c^2=b^2-2ac$	1
(ii) • substitution in expansion of $(a+c)^2$ to show $(a+c)^2=a^2+b^2$	1

Answer



(i)

$\Delta ABN \parallel\!| \Delta ACB$ (*given*)

$$\frac{BN}{CB}=\frac{AB}{AC}=\frac{AN}{AB}\quad\left(\begin{array}{c}\textit{corresponding sides of similar}\\ \Delta\textit{'s are in proportion}\end{array}\right)$$

$$\frac{d}{2a}=\frac{c}{b}=\frac{b-d}{c}\Rightarrow\begin{cases}bd=2ac\\ c^2=b(b-d)=b^2-bd\end{cases}$$

$$\therefore\ c^2=b^2-2ac$$

(ii)

$$\left.\begin{array}{l}b^2=c^2+2ac\quad (\textit{from } (i))\\ (a+c)^2=a^2+c^2+2ac\end{array}\right\}\Rightarrow (a+c)^2=a^2+b^2$$

(b) **Outcomes Assessed:** (i) **PE2, PE3** (ii) **PE3**

Marking Guidelines

Criteria	Marks
(i) • establishing that $P(0)$, $P(1)$ have opposite signs	1
• noting that $P(x)$ is continuous to deduce existence of root α, $0<\alpha<1$	1
(ii) • quoting correct expression for approximate value of α using Newton's method	1
• calculating approximate value of α correct to 2 decimal places	1

Answer

(i) $P(x)=x^3+3x^2+6x-5 \Rightarrow \begin{cases}P(0)=-5<0\\ P(1)=\ \ 5>0\end{cases}$ and $P(x)$ is continuous

$\therefore\ P(x)=0$ has a root α, $0<\alpha<1$.

(ii) $P'(x)=3x^2+6x+6$ $\qquad \alpha \approx 0\cdot5-\frac{P(0\cdot5)}{P'(0\cdot5)}=0\cdot5-\frac{(-1\cdot125)}{9\cdot75}\approx 0\cdot62$ (to 2 decimal places)

(c) **Outcomes Assessed:** **HE6**

Marking Guidelines

Criteria	Marks
• using substitution process correctly to obtain new integrand in terms of u	1
• finding the new limits for the integral in terms of u	1
• obtaining the primitive function $2\sin^{-1}u$	1
• evaluating the definite integral by substitution of the limits	1

Answer

$x = u^2 \quad u > 0$

$dx = 2u\,du$

$x = \frac{1}{4} \Rightarrow u = \frac{1}{2}$

$x = \frac{1}{2} \Rightarrow u = \frac{1}{\sqrt{2}}$

$$I = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx = \int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{1}{u\sqrt{1-u^2}}\,2u\,du$$

$$I = 2\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-u^2}}\,du = 2\left[\sin^{-1}u\right]_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}}$$

$$I = 2\left(\sin^{-1}\tfrac{1}{\sqrt{2}} - \sin^{-1}\tfrac{1}{2}\right) = 2\left(\tfrac{\pi}{4} - \tfrac{\pi}{6}\right) = \tfrac{\pi}{6}$$

Question 4

(a) **Outcomes Assessed:** (i) **H5** (ii) **H8**

Marking Guidelines

Criteria	Marks
(i) • obtaining the primitive function $\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)$	1
• evaluation of the definite integral by substitution of the limits	1
(ii) • using the pattern for Simpson's rule with correct x values, h value and multipliers.	1
• calculation of 3 function values and final approximation for definite integral	1

Answer

(i)

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x\,dx$$

$$= \tfrac{1}{2}\int_0^{\frac{\pi}{2}} (1 - \cos 2x)\,dx$$

$$= \tfrac{1}{2}\left[x - \tfrac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$$

$$I = \tfrac{1}{2}\left(\tfrac{\pi}{2} - 0\right) = \tfrac{\pi}{4}$$

(ii)

$f(x) = \sin^2 x, \quad h = \frac{\pi}{4}$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
f	0	$\frac{1}{2}$	1
	×1	×4	×1

$$I \approx \tfrac{h}{3}\{f_0 + 4f_1 + f_2\}$$

$$= \tfrac{\pi}{12}\{0 + 2 + 1\}$$

$$= \tfrac{\pi}{4}$$

4(b) **Outcomes Assessed:** (i) **PE3** (ii) **PE3**

Marking Guidelines

Criteria	Marks
(i) • determining that there are 3 appropriate sets of three cards for a sum of 9	1
• calculating $\frac{3}{{}^9C_3} = \frac{1}{28}$ as the required probability	1
(ii) • realising that there are now ${}^8C_2$ possible sets of three cards given 2 is selected	1
• calculating $\frac{2}{{}^8C_2} = \frac{1}{14}$ as the required probability	1

Answer

(i) Exactly 3 sets of cards have a sum of 9: 1+2+6, 1+3+5, 2+3+4

$$\therefore P(\textit{sum is } 9) = \frac{3}{{}^9C_3} = \frac{3.3.2.1}{9.8.7} = \frac{1}{28}$$

(ii) If the set of cards contains the number 2, exactly two such sets have a sum of 9.
The two cards chosen to complete the set of 3 are selected from the remaining 8 cards.

$$\therefore P(\textit{sum is 9 given first card is } 2) = \frac{2}{{}^8C_2} = \frac{2.2.1}{8.7} = \frac{1}{14}$$

4(c) **Outcomes Assessed: PE2, HE5**

Marking Guidelines

Criteria	Marks
• finding the relationship between $\frac{dV}{dt}$ and $\frac{dr}{dt}$	1
• finding the relationship between $\frac{dL}{dt}$ and $\frac{dV}{dt}$, where the equator has length L cm	1
• using the numerical values of $\frac{dV}{dt}$ and r to show $\frac{dL}{dt}=0\cdot125$	1
• interpreting this to deduce that length of equator is increasing at a rate of $0\cdot125\text{cm s}^{-1}$	1

Answer

$V=\frac{4}{3}\pi r^3$ $\qquad$ $L=2\pi r$

$\frac{dV}{dt}=\frac{4}{3}\pi.3r^2\frac{dr}{dt}$ $\qquad$ $\frac{dL}{dt}=2\pi\frac{dr}{dt}=2\pi.\frac{1}{4\pi r^2}\frac{dV}{dt}$

$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$ $\qquad$ $\therefore \frac{dL}{dt}=\frac{1}{2r^2}\frac{dV}{dt}=\frac{25}{2\times10^2}=0\cdot125$ $\quad$ when $r=10$

Length of equator is increasing at a rate of $0\cdot125\text{cm s}^{-1}$ when the radius is 10 cm

Question 5

(a) **Outcomes Assessed:** (i) **HE4** (ii) **P5, HE4**

Marking Guidelines

Criteria	Marks
(i) • finding the equation of the inverse function $f^{-1}(x)$	1
(ii) • showing intercepts on the coordinate axes and asymptotes for both curves	1
• showing intersection point $(1, 1)$	1
• correct shapes with curves as reflections in $y=x$	1

Answer

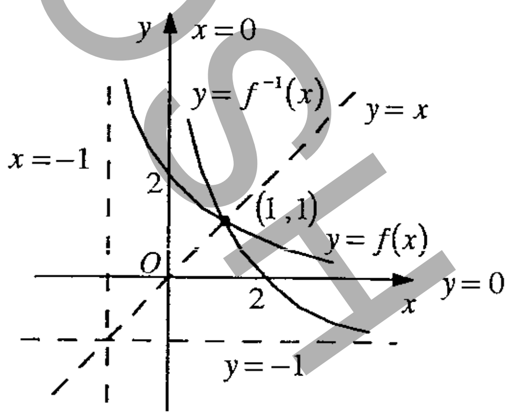
(i)

$y=\frac{2}{x+1}\Rightarrow(x+1)=\frac{2}{y}\Rightarrow x=\frac{2}{y}-1$

$f(x)=\frac{2}{x+1},\ x>-1\Rightarrow f^{-1}(x)=\frac{2}{x}-1,\ y>-1$

$\therefore f$ has inverse $\quad f^{-1}(x)=\frac{2}{x}-1,\ x>0$

Curves are reflections in $y=x$ and hence intersect on $y=x$ where $\frac{2}{x+1}=x\Rightarrow x=1$



(ii) $\tan^{-1}3x+\tan^{-1}2x=\frac{\pi}{4} \Rightarrow \dfrac{3x+2x}{1-3x.2x}=\tan\frac{\pi}{4}$, using $A=3x,\ B=2x$ in (i)

$\dfrac{5x}{1-6x^2}=1 \Rightarrow \begin{aligned}&6x^2+5x-1=0\\&(6x-1)(x+1)=0\\&x=\tfrac{1}{6} \text{ or } x=-1\end{aligned}$

But $x=-1 \Rightarrow \begin{cases}\tan^{-1}3x<0 \text{ and } \tan^{-1}2x<0\\ \therefore \tan^{-1}3x+\tan^{-1}2x\neq\frac{\pi}{4}\end{cases}$

Hence $x\neq-1$. $\therefore x=\frac{1}{6}$

6(b) **Outcomes Assessed:** (i) **H3, HE3** (ii) **H3, HE3**

Marking Guidelines

Criteria	Marks
(i) • finding value of A	1
• finding exact value of k	1
(ii) • showing $t=\dfrac{\ln 2}{k}$	1
• finding the further time 5 min 38 s	1

Answer

(i) $T=20+Ae^{-kt}$

$\left.\begin{aligned}t&=0\\T&=100\end{aligned}\right\} \Rightarrow \begin{aligned}100&=20+Ae^0\\100&=20+A\end{aligned}$ $\therefore A=80$ and $T=20+80e^{-kt}$ Then

$\left.\begin{aligned}t&=4\\T&=80\end{aligned}\right\} \Rightarrow \begin{aligned}&80=20+80e^{-4k}\\&e^{-4k}=\tfrac{60}{80}=\tfrac{3}{4}\end{aligned}$ $\therefore \begin{aligned}-4k&=\ln\tfrac{3}{4}\\k&=-\tfrac{1}{4}\ln\tfrac{3}{4}=\tfrac{1}{4}\ln\tfrac{4}{3}\end{aligned}$

(ii)

$\left.\begin{aligned}T&=20+80e^{-kt}\\T&=60\end{aligned}\right\} \Rightarrow \begin{aligned}e^{-kt}&=\tfrac{40}{80}=\tfrac{1}{2}\\-kt&=\ln\tfrac{1}{2}=-\ln 2\end{aligned}$ $\therefore t=\dfrac{\ln 2}{\left(\frac{1}{4}\ln\frac{4}{3}\right)}\approx 9\cdot 6377$

Hence it falls to 60°C after 9 min 38 sec, that is after a further 5 min 38 sec.

6(c) **Outcomes Assessed:** (i) **PE2, HE3** (ii) **H5, HE3**

Marking Guidelines

Criteria	Marks
(i) • finding values of v and a when $t=0$	1
• interpreting these values to deduce particle is moving right and slowing down	1
(ii) • showing if particle is at O at time t, then $\tan 2t=-3$	1
• solving this equation to find the first such time.	1

Answer

(i)

$x=3\cos 2t+\sin 2t$ $\therefore t=0 \Rightarrow x=3,\ v=2,\ a=-12$

$v=-6\sin 2t+2\cos 2t$ Hence particle is initially 3 m to the right of O,

$a=-12\cos 2t-4\sin 2t$ moving to the right (since $v>0$) and

$a=-4x$ slowing down (since v and a have opposite signs)

5(b) **Outcomes Assessed:** (i) **HE5** (ii) **H5, PE2**

Marking Guidelines

Criteria	Marks
(i) • obtaining expression for a in terms of x	1
(ii) • integrating expression for $\frac{dt}{dx}$ to obtain primitive function (even if $+c$ omitted)	1
• including and evaluating the constant of integration to find t in terms of x	1
• finding x in terms of t by rearrangement.	1

Answer

(i) $v = -x^2 \Rightarrow a = v\frac{dv}{dx} = -x^2 . (-2x) = 2x^3$

(ii)

$$\frac{dx}{dt} = -x^2 \Rightarrow \frac{dt}{dx} = -\frac{1}{x^2} \Rightarrow t = \frac{1}{x} + c, \quad c \text{ constant}$$

$$\left.\begin{array}{l} t=0 \\ x=1 \end{array}\right\} \Rightarrow 0 = 1 + c \Rightarrow c = -1 \Rightarrow t = \frac{1}{x} - 1 \quad \therefore \quad x = \frac{1}{t+1}$$

5(c) **Outcomes Assessed:** **PE3**

Marking Guidelines

Criteria	Marks
• writing general term with appropriate binomial coefficient and powers of x^2 and $\frac{a}{x}$	1
• showing term independent of x is ${}^6C_4 a^4$ or ${}^6C_2 a^4$	1
• deducing ${}^6C_4 a^4 = 240$ or ${}^6C_2 a^4 = 240$ and hence $a^4 = 16$	1
• stating both solutions $a = \pm 2$	1

Answer

General term in expansion of $\left(x^2 + \frac{a}{x}\right)^6$ is ${}^6C_r \left(\frac{a}{x}\right)^r \left(x^2\right)^{6-r} = {}^6C_r\, a^r\, x^{12-3r}$, $r = 0, 1, 2, ..., 6$

Then term independent of x is ${}^6C_4\, a^4\, x^0 = 15\, a^4 \Rightarrow 15\, a^4 = 240 \Rightarrow a^4 = 16 \quad \therefore\ a = \pm 2$

Question 6

6(a) **Outcomes Assessed:** (i) **H5, HE4** (ii) **P4, HE7**

Marking Guidelines

Criteria	Marks
(i) • showing $\tan\theta = \frac{A+B}{1-AB}$	1
(ii) • showing $6x^2 + 5x - 1 = 0$	1
• solving this quadratic equation	1
• rejecting the solution $x = -1$ with explanation	1

Answer

(i) Let $x = \tan^{-1} A$ and $y = \tan^{-1} B$. Then $\theta = x + y$, $\tan x = A$, $\tan y = B$ and hence

$$\tan\theta = \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x . \tan y} = \frac{A+B}{1-AB}$$

(ii) At O, $x = 0$

$$3\cos 2t + \sin 2t = 0$$
$$\sin 2t = -3\cos 2t$$
$$\tan 2t = -3$$

smallest positive such t is given by

$2t = \pi - \tan^{-1} 3 \;\Rightarrow\; t = \frac{1}{2}\left(\pi - \tan^{-1} 3\right) \approx 0{\cdot}95$

$\therefore$ particle first reaches O after $0{\cdot}95$ s (to 2 dec. pl.)

Question 7

7(a) **Outcomes Assessed:** (i) **P5, PE6** (ii) **P5, PE6, HE2** (iii) **P5, PE2, PE6**

Marking Guidelines

Criteria	Marks
(i) • showing $f(0) = 1$	1
• showing $f(-x) = \dfrac{1}{f(x)}$	1
(ii) • noting that $S(1)$ is true	1
• showing that if $S(k)$ is true, then $S(k+1)$ is true	1
• deducing the truth of $S(n)$ for all positive integers	1
(iii) • using (i) and (ii) to deduce that $f(-nx) = [f(x)]^{-n}$	1

Answer

(i)

$$f(0+0) = f(0)\,.\,f(0)$$
$$f(0) - f(0)\,.\,f(0) = 0$$
$$f(0)\,[1 - f(0)] = 0$$
$$\therefore f(0) > 0 \Rightarrow f(0) = 1$$

$$f(x + [-x]) = f(x).\,f(-x)$$
$$\therefore f(x).\,f(-x) = f(0) = 1$$
$$\therefore \quad f(-x) = \frac{1}{f(x)}$$

(ii) Let $S(n)$ be the statement $f(nx) = [f(x)]^n$, $n = 1, 2, 3, \ldots$

Clearly $S(1)$ is true, since $f(1.x) = [f(x)]^1$.

If $S(k)$ is true for some positive integer k, then $f(kx) = [f(x)]^k$ **

Consider $S(k+1)$:

$$f([k+1]x) = f(kx + x) = f(kx).\,f(x)$$
$$= [f(x)]^k.\,f(x) \quad \text{if } S(k) \text{ is true, using **.}$$
$$= [f(x)]^{k+1}$$

Hence if $S(k)$ is true for some positive integer k, then $S(k+1)$ is true. But $S(1)$ is true. Hence $S(2)$ is true, and then $S(3)$ is true and so on. Hence $S(n)$ is true for all positive integers n.

(iii) If n is a positive integer, $f(-nx) = \dfrac{1}{f(nx)} = \dfrac{1}{[f(x)]^n}$, using (i) and (ii), and hence $f(-nx) = [f(x)]^{-n}$.

7(b) **Outcomes Assessed:** (i) **HE3** (ii) **PE2, HE3**

Marking Guidelines

Criteria	Marks
(i) • writing expressions for horizontal displacements of both particles	1
• writing expressions for vertical displacements of both particles	1
(ii) • showing $U\cos\alpha = V\cos\beta$	1
• showing $UT\sin\alpha = h + VT\sin\beta$	1
• eliminating V from this relationship	1
• rearrangement to obtain T in required form	1

Answer

(i) For particle projected from O

$x_O = Ut\cos\alpha$

$y_O = Ut\sin\alpha - \frac{1}{2}gt^2$

For particle projected from A

$x_A = Vt\cos\beta$

$y_A = h + Vt\sin\beta - \frac{1}{2}gt^2$

(ii) Particles collide at time T, having equal horizontal displacements and equal vertical displacements.

$$UT\cos\alpha = VT\cos\beta \Rightarrow U\cos\alpha = V\cos\beta \quad (1)$$

$$UT\sin\alpha - \tfrac{1}{2}gT^2 = h + VT\sin\beta - \tfrac{1}{2}gT^2 \Rightarrow UT\sin\alpha = h + VT\sin\beta \quad (2)$$

From (2): $T(U\sin\alpha - V\sin\beta) = h$

$T(U\sin\alpha\cos\beta - V\cos\beta\sin\beta) = h\cos\beta$

Using (1): $T(U\sin\alpha\cos\beta - U\cos\alpha\sin\beta) = h\cos\beta$

$UT\sin(\alpha-\beta) = h\cos\beta$

$$\therefore \quad T = \frac{h\cos\beta}{U\sin(\alpha-\beta)}$$